**page 1**: **1**. 97.1 mi **2**. base area times height **3**. 72 **4**. 144 bars; 24 bars **5**. 0.430 lap = 108.0 m along arc; 78.1 m along chord **6**. 1; Pythagorean **7**. $2400(1+\sqrt{2}) = 5794$ cu ft **8**. 5.91 cu in; no **9**. 44% **10**. 38.5%; 35.9%; 34.3%; 33.8%; approaches 1/3

**page 2**: **1**. $[-3\sqrt{3}, \pm 3]$ **2**. solar eclipses **3**. they are congruent and dissect the cube; each has one third the volume of the cube **4**. 140.2 deg **5**. $[-6.71, \pm 9.95]$ **6**. 210 mi **7**. 14 wedges; 491 cu in **8**. 9389520 tons **9**. cos 120 = –0.5 and sin 120 = 0.866 **10**. $TF = 23.0$ cm; vol = 3067 cc **11**. angle $TAF = \sin^{-1}(23/27) = 58.4$ deg is smallest; angle $TMF = \tan^{-1}(23/10) = 66.5$ deg is largest, where $M$ is the midpoint of $AB$ **12**. $KLMN$ is a square, with $KL = 40/3$ cm

**page 3**: **1**. 73.7 in **2**. 85.7 in **3**. (–3, –1); (15, 5); use triangle inequality **4**. two angles are $\cos^{-1}(0.5m/p)$; one is $2\sin^{-1}(0.5m/p)$ **5**. $1536\sqrt{3} = 2660$ cc; 1545 sq cm **6**. (5, 5), 5; (17, 17), 17 **7**. let $CD = 2$: $CIEJ$ is a rhombus, $CI = \sqrt{5}$; its area is $2\sqrt{6} = 4.90$; area $BDHF$ is $4\sqrt{2} = 5.66$ = largest slice? **8**. 75 deg **9**. $(\cos\theta, \sin\theta)$ **10**. 355 ft; 7819 sq ft **11**. 0.5 deg **12**. use common tangent **13**. $(\cos 36t, \sin 36t)$

**page 4**: **1**. 17.0 in **2**. 45.2 deg **3**. 64/125 and 61/125 **4**. after 11.46 laps: (–96.7,25.4) **5a**. 26132.116; **5b**. $0.5h^2 \cos A \sin A$ **5c**. 26132.116 **6**. $P$ = (cos 215, sin 215) = (–0.819, –0.574) **7**. center (1, 2) and $r = 3$; center (1, 2) and $r = 0$; none **8**. a circle

**page 5**: **1**. (–1,3); $H(x, y) = (6 - x, 4 - y)$ **2**. 14371 sq ft **3**. 106.3 deg; 18.546 in **4**. unit circle **5**. $\frac{1}{2}rs$ **6**. twice as likely to land touching a line **7**. $(3+\cos\theta, 2+\sin\theta)$ **8**. (3.091, 5.954) = $(\sqrt{45}\cos 62.57, \sqrt{45}\sin 62.57)$ **9**. 101 ft **10**. 54 sq in; 43.0° **11**. $r$=4.5, $s$=24, $\theta = 305.6°$ **12**. cos 240 = –1/2; sin 240 = $-\sqrt{3}/2$

**page 6**: **1**. cos 100 = cos 260 = sin 190 = –0.173648; cos 280 = 0.173648 **4**. (–1, 7); $H(x,y) = (2a - x, 2b - y)$ **5**. $(x-3)^2 + (y+5)^2 = 49$ **6**. center (1,5); mag = 3 **7**. $y = 5$; $4x+3y = 19$; 53.1° **8a**. $w = h\cos B$; $x = h\cos B\cos A$; $y = h\cos B\sin A$ **8b**. $z = h(\cos B)^3$ **9**. $\theta = 60$, $\theta = 120$ **10**. 1 **11**. unit circle, centered at (2, 1) **12**. $(x,y)$ = (2.638, 6.347) **13**. (–7, 24) **14a**. $x^2y^2$ **14b**. $x^2 + 2xy + y^2$ **14c**. $a^2 \sin^2 B$ **14d**. $a^2 + 2a\sin B + \sin^2 B$

**page 7**: **1**. half-turn at (2,7) **2a**. 180 **2b**. 70, 290 **2c**. 225, 315 **2d**. 81, 261 **3**. 7873 sq ft **4**. 135 ft **5**. $(x,y)$ =(3.362, 0.653) **6**. $(b, a)$; $((a+b)/2, (a+b)/2)$ **7**. arc is $\theta\pi r/180$; perimeter is $2r + \theta\pi r/180$ **8**. $(x, y) = (5, 6) + t[2, 3]$ **9**. 65% **10**. both have radius 3 **11**. (4, 6); $(5-y, x-1)$ **12**. incenter

**page 8**: **1a**. $x$ **1b**. $2x$ **2**. 16.8 cm **3**. 91.3 sq cm **4a**. $CD = a\sin B$ **4b**. $BD = a\cos B$ **4c**. $a^2$ **4d**. clarity **5**. 228 mm **6**. 91.3 sq cm **7**. radius = 1/2 in, volume = 0.544 cu in **8a**. 405 deg, 585 deg **8b**. 463 deg, 643 deg **8c**. 610, 650 **9**. the center is $(a, -2b)$, the radius is $\sqrt{a^2+4b^2}$ **10**. 16.1 mph; 1200 deg/sec **11**. same formula for both **12a**. 251, 109 **12b**. no solution **12c**. 220, 320 **12d**. 250, 110 **13**. no; $x^2+y^2+1=0$ is a counter example; $m^2+n^2+4p$ has to be positive **14**. $30 < t < 330$

**page 9**: **1**. $T$ is a counterclockwise 36.87-degree rotation; the points seem not to repeat **2**. 23.3 m **3**. $100\pi = 314$ cc **4**. 12.5% **5**. $r$=13, $s$=$10\pi$, $\theta$=138.5 deg **6**. 3'8" **7a**. [–8,12,4] **7b**. [–2, 4, 3] **7c**. [–8,11,2] **7d**. 8 **7e**. 8 **8**. 33/14, 11/2 **9**. (60,37,0) **10**. 10.7 mph **11**. 4/9 = 44.4% **12**. $\{\text{arc,diameter}\} = \{16.52, 12.84\}$ **13**. only one sector if arclength equals diameter, or if diameter more than $\pi$ times the arclength

**page 10**: **1**. $y = (a^2+b^2-x^2)/2ab$ **2**. $c = \sqrt{a^2+b^2-2ab\cos C}$ **3**. $(1/2)ab\sin C$ **4**. 16 in **5**. $12/(\cos\theta\sin 2\theta)$ **6**. 35.3 **7**. 23.6 deg *or* 156.4 deg **8**. 88.6 sec **9.** circle parametrized according to previous story **10**. $(x, y) = (-3+4\cos\theta, 2+4\sin\theta)$ or $(x, y) = (-3+4\sin\theta, 2+4\cos\theta)$, etc **11b**. square of the length **11d**. yes **11e**. a rhombus is formed by placing two vectors of the same length tail to tail; [4,7,4] and [1,4,8], for example

**page 11**: **1**. $h = \sqrt{128}$ = 11.3 cm; $V$ = 189.6 cc **2**. 5.02 in **3a**. 340, 20 **3b**. 70, 290 **3c**. 80, 100 **3d**. 210< $w$ <330 **3e**. 45< $w$ <90 or 225 < $w$ <270 **4**. 22.6 cm **5**. $2r\sin\beta$ **6**. $Q(x, y) = (-y, x)$; i.e., $a = 0$, $b = -1$, $c = 1$, $d = 0$ **8**. 0 meters < $c$ < 2 meters; 0 meters < $s$ < 2π meters **9**. $c = 2\sin(90s/\pi)$ or $c = \sqrt{2-2\cos(180s/\pi)}$ **10**. the square of the length of **u** **11**. yes **12**. 4.34 in

**page 12**: **1**. 5.37 in **2**. 2.89, 3.0, 2.67 **3**. $r = p\sqrt{100-p^2}/(p+10)$ or $r = p\tan((1/2)\cos^{-1}(p/10))$, which is greatest when $r$ = 3.003 and $p = -5+\sqrt{125}$ = 6.180 **4**. 15.3 cm **5**. they all equal the diameter of the circumscribed circle **6**. 5, 17, 19.65, –12; 7, 3, 5.66, 4 **7**. $60\pi$ = 188.5 sq in **8**. 2.52 km **9**. the SSS version is $\cos C = (a^2+b^2-c^2)/(2ab)$; 41.4 deg, 55.8 deg, 82.8 deg **10**. 5, [4.045,2.939] **11a**. 120-deg sector has $V$ = 189.6 cc; 180-deg sector has $V$ = 391.8 cc; **11c**. 294-deg sector has $V$ = 696.5 cc; in general, $V = (\pi\theta^2/2700)\sqrt{144-(\theta/30)^2}$

**page 13**: **1**. **u** is perpendicular to **v**; this is the Pythagorean Theorem **3**. the dot products of **P** with the rows of **S** are 2320.50, 1791.00, and 1626.00 **5a**. 17.32 sq in **5b**. 4 in **5c**. 7 in **5d** $Q$=38.2 deg and $R$=81.8 deg **5e**. 4.58 in **5f**. 4.41 in **5g**. 120-deg angle between the given sides, and opp side 11.36 in **6**. $\tan\theta$ **7**. 15.2 in, 11.3 in **8**. [2,–4,–3] represents third side; 14 and 29 are squares of lengths

**page 14**: **1**. Law of Sines **2**. New England Pizza: \$94.50, Romeo's: \$100.25, Supreme: \$101.00 **3**. New England Pizza: \$64.00, Romeo's: \$63.00, Supreme: \$68.50 **4**. the cosines of supplementary angles are opposites **5**. $\sin 2\alpha = 2\sin\alpha\cos\alpha$ **6**. 8 inches **7**. 0; the parallelogram identity says $a^2 + b^2 + a^2 + b^2 = c^2 + d^2$, where $a$ and $b$ are the sides and $c$ and $d$ are the diagonals **8a**. $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$ **8b**. $\begin{bmatrix} 15 & 12 \\ -27 & 20 \end{bmatrix}$ **8c**. $\begin{bmatrix} -46 \end{bmatrix}$ **9**. latitude circle 32.13 deg north **10a**. 98.13 deg **10b**. 120 deg **11**. try an isosceles right triangle; it is never true **12**. 17 in or $\sqrt{73}$ in

**page 15**: **1**. 1.63 m; 1.63 m; 6 – 5 cos $t$ **3b**. $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| \cdot |\mathbf{v}| \cos\theta$ **4**. 4021 sq ft **5**. **MM** $= \begin{bmatrix} a^2 + bc & ab + bd \\ ca + cd & bc + d^2 \end{bmatrix}$, **MN** = $\begin{bmatrix} b \\ d \end{bmatrix}$, **PM** = $\begin{bmatrix} a & b \end{bmatrix}$, **NP** = $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, **PN** = [0] **6**. 59.5 deg; $\begin{bmatrix} -165/169 & 396/169 \end{bmatrix}$ **7**. 36.9 deg and 143.1 deg are supplementary **8**. 143.1 deg; −216.9 deg **9**. $n$ must equal $p$ **10**. $(x-2)^2 + (y-3)^2 + (z+6)^2 = 25$; no, $5 < 6$

**page 16**: **1**. 13510 sq ft or 29444 sq ft **2**. a sphere **3a**. $Q$ = 49.2 deg or 130.8 deg **3b**. only $Q$ = 31.6 deg **3c**. no such triangle **3d**. only $Q$ = 90 deg **4**. $h = r - r\cos(2160d/\pi r)$ **5**. wheel turns $2160/\pi r$ deg per foot rolled **6a**. half-turn about (0,0) with magnification factor of 3 **6b**. counterclockwise quarter-turn about (0,0) **6c**. reflection across $y = -x$ **6d**. 53.13-deg counterclockwise rotation about (0,0) **7b**. $a = d = 0$, $b = -1$, $c = 1$ **7c**. $a = d = 0$, $b = c = -1$ **7d**. $a = d = .6$, $b = -0.8$, $c = 0.8$ **8a**. $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ **8b**. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **8c**. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **8d**. $\begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$ **9**. 48.2 deg; [52/9,52/9,26/9] **10**. the circle $y = 0$, $(x-2)^2 + (z+6)^2 = 16$

**page 17**: **1**. $h = \sin t$ **2**. $p = \cos t$ **3a**. 53 sec, 127 sec; 233 sec, 307 sec **3b**. 60 sec, 300 sec; 180 sec **4**. sin $\theta$, –cos $\theta$ **5**. –sin $\theta$, –cos $\theta$, tan $\theta$ **6**. 35 deg **7**. 2972 mi; 18197 mi

**page 18**: **1b**. the first two products are just the columns, which are the images of [1,0] and [0,1] **2**. dist =213 m **3**. cos $A$ is 0.28 or –0.28; cos $A$ is –0.28 **4**. the projection is [1.92,−2.56] **5**. 82.375 **6**. 30.8 km **7**. 84 sq in **8a**. dilation about (0,0) with magnification factor of 3 **8b**. half-turn about (0,0) **8c**. 67.38 deg counterclockwise rotation about (0,0) **8d**. 53.13 deg counterclockwise rotation about (0,0) with magnification factor of 5 **9**. 8b and 8c are isometries **10**. the triangle inequality: the sum of two sides of a triangle is at least as large as the third side

**page 19**: **1**. $x = 4.22$, $t = 1.32$ hr **2**. $\sin PJO = 0.4665$, $\sin PDN = 0.9330$; at 15 uph, $x = 6.82$, $\sin PJO = 0.6489$, $\sin PDN = 0.8652$ **3**. $\begin{bmatrix} \cos 57 & -\sin 57 \\ \sin 57 & \cos 57 \end{bmatrix}$, yes **4**. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **5**. (3.301, 2.934, 4.401) **6**. $P = (-42, 0.7431)$, $R = (318, 0.7431)$, and $S = (402, 0.7431)$ **7**. 45 deg **8**. (0,5); same angle **9**. circle of radius $\sqrt{65}$ centered at the origin; it has the given points at opposite ends of a diameter

**page 20**: **1**. 22.6 deg; (144/169)[–3,4,12] **2**. area = $\pi re$ **3a**. 54.0 deg, 126.0 deg, and 414.0 deg **3b**. 144.0 deg, 216.0 deg, and 504.0 deg **3c**. 63.0 deg, 243.0 deg, and 423.0 deg **4b**. **w** and **u** point in opposite directions; yes **5**. $m = 0.754$, $m = -0.933$, $m = 1.540$, $m = \tan t$, $m$ = the slope of $OT$ **6**. $t = 0$, $t = 26.6$, $t = 116.6$, $m$ can be any number, but $t$ can not be 90 or 270 **7a**. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **7b**. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **7c**. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is $x$-axis reflection **7d**. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is $y$-axis reflection **7e**. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **8**. $Q = (180-\theta, k)$, $R = (360+\theta, k)$, $S = (540-\theta, k)$

**page 21**: **1**. the Angle Bisector Theorem: $p/q = a/c$ **2**. 2.83 in **3**. $r = 9.43$ and $\theta = 122.0$ **4**. rotation matrices **5a**. [4,0,0] = **AB**; [0,6,0] = **AD**; [0,0,3] = **AH** **5b**. $\left(\frac{44}{13}, \frac{66}{13}, 0\right)$ **6**. $\begin{bmatrix} \cos 240 & \cos 150 \\ \sin 240 & \sin 150 \end{bmatrix} = \begin{bmatrix} \cos 240 & \sin 240 \\ \sin 240 & -\cos 240 \end{bmatrix}$, $\begin{bmatrix} -1.366 & -0.366 \end{bmatrix}$ **7**. the sphere with diameter $AB$ **8**. 5 in by 10 in **9**. $\cos\theta$, $-\sin\theta$, $\cos\theta$, $\sin\theta$, $\tan\theta$ **10**. $\sqrt{r^2 - d^2}$

**page 22**: **1**. the product is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; identity transformation **2**. area 30.6 sq in; diags 6.13 in and 11.85 in **3**. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **4**. diameter = 169/12 = 14.1 in **5**. triangle is a segment of length $p+q$ **6**. all products **MN** and **NM** are identity matrices **7**. $\begin{bmatrix} 1/4 & 0 \\ 0 & 1/3 \end{bmatrix}$; $\begin{bmatrix} 3/8 & 2/8 \\ -1/8 & 2/8 \end{bmatrix}$ **8a**. $r = 1$, $\theta = 90$ deg **8b**. $r = 1.414$, $\theta = 135$ deg **8c**. $r = 5$, $\theta = 323.1$ deg **8d**. $r = 7.07$, $\theta = 81.9$ deg **8e**. $r = 7.07$, $\theta = 261.9$ deg **9**. 40.5 deg, 19.5 deg **10**. 27/64 the depth of the cylinder; 3/4 the depth of the cone **11**. 10.31 in

**page 23**: **1**. 12441 miles **2**. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ and $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ **3**. 22 ft 2 in **4**. (5, 4, 3); 60 deg **5**. both are 0.618 **6**. 45, 225, .... **7**. 25.55 **8**. great-circle arc is 55.4-deg; arc length 12.57 **9**. 12.2 in **10**. the 1-by-2 matrix **PS** = [13550 9145] shows monthly sales; **SP** makes no sense **11**. multiply **P** by 0.90

**page 24**: **1**. $25\pi$ **2**. the shaded areas are both $\pi h^2$ **3**. the cone volume is one third of the cylinder volume; the hemisphere volume is therefore two thirds of the cylinder volume **5**. \$13, \$9, \$23 **6**. $\sin\theta/\cos\theta$, *or* $\tan\theta$ **7**. north of the latitude circle 77.3 deg N **8**. $180-\theta$; $P = (\cos\theta, \sin\theta)$; $Q = (-\cos\theta, \sin\theta)$ **9**. $\sin\theta$; $-\cos\theta$ **10**. $5625\pi = 17671$ cc

**page 25**: **1**. 4 :5 ; most of the volume is near the top **2**. 66 2/3 % **3**. 6220 mi; 4147 mi **4**. triangle is a segment of length $|p-q|$ **5**. 0.17 cm **6a**. $\begin{bmatrix}5 & 3\\ 4 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ 1\end{bmatrix}$ **6b**. $\begin{bmatrix}2 & 0 & -5\\ 0 & 3 & 4\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-8\\ 17\\ 6\end{bmatrix}$ **6c**. $\begin{bmatrix}2 & 1\\ -5 & -3\end{bmatrix}\begin{bmatrix}t\\ u\end{bmatrix}=\begin{bmatrix}-1\\ 1\end{bmatrix}$ **7a**. $\begin{bmatrix}15\\ 1\end{bmatrix}$ **7b**. $\begin{bmatrix}-8\\ 17\\ 6\end{bmatrix}$ **7c**. $\begin{bmatrix}-1\\ 1\end{bmatrix}$ **7d**. $\begin{bmatrix}-7\\ 12\\ 3\end{bmatrix}$

**page 26**: **1**. 65/8 = 8.125 **2**. $360-\theta$; $P = (\cos\theta, \sin\theta)$; $Q = (\cos\theta, -\sin\theta)$ **3**. $-\sin\theta$ and $\cos\theta$ **4**. 3110 mi; 1466 mi **5**. (45, 0.707); (225, –0.707); (405, 0.707) **6**. no; unless the matrices are square, the two products are not even the same size; the relation is true at least when **M** and **N** are inverses **7**. $r = 8.06$, $\theta = -60.3$ or 299.7 **8**. $36\pi$; $\pi/6 = 52.4\%$ **9.** crust vol = 1965635880 cu mi; surface area is somewhat larger than the estimate 196563588 sq mi **10**. see item 2 **11**. 301.6 cc = 127.2 cc + 174.4 cc **12**. as shown in items **6** and **7d** on preceding page, $[x, y, z] = [2, 3, 1]$

**page 27**: **1**. $(10\cos 40\cos 33, 10\cos 40\sin 33, 10\sin 40) = (6.42, 4.17, 6.43)$ **2**. $x$ = 23.0, 157.0, and 383.0 **3**. $x$ = 67.0, 293.0, and 427.0 **4**. clockwise 120° rotation about (0,0) **5**. $x$ = 8.00, $y$ = 5.15 **6a**. (–3, 5) **6b**. (3, –5) **6c**. (–3, 5) **7a**. [8, –13] **7b**. [1.805, –0.439, 0.854] **7c**. [5.638, 1.258, 0.882] **8**. $60\sin 36 = 35.3$ cm **9**. $\cos 40$; $\sin 50$ **10**. 8 in **11**. 48.2 deg

**page 28**: **1**. $\sqrt{5} = 2.24$ in **2**. rotation matrix for angle $\alpha+\beta$ **3**. no; cos is not linear **4**. 758 mph; $1037\cos L$ **5**. $-5/13$ and $-12/5$ **6**. $\cos\theta = \sqrt{b^2-a^2}/b$ and $\tan\theta = a/\sqrt{b^2-a^2}$ **7**. $5^9$; $b^x b^y = b^{x+y}$ **8**. 1 unit **9a**. $30\times10^{14}$ or $3\times10^{15}$ hairs **9b**. $9\times10^{15}$ meters **10**. $1037\cos\theta$ **11**. 4.516 cu in; 451.6 cu in; actual area is 452.4 sq in

**page 29**: **1b**. $G$ = (2492, 0, 3077) **1c**. $E$ = (3321, 2157, 0) **1d**. $A$ = (2544, 1652, 2545) **2**. 25.33° great-circle arc $GA$ is 1751 miles long **3**. yes; $\theta$ = 63.4 and 243.4, for example **4**. $y = (5/24)x^2 + (19/24)$ **5**. $r$ = 6.93 cm; 4/9 occupied **6**. common period and $x$-intercepts, but different amplitudes; graph enters fourth quadrant from the origin **7**. $x^3 - y^3$ **8**. $\frac{4}{3}\pi(R^2+Rr+r^2)(R-r)$ is the crust volume; $R-r$ is the thickness; $R^2+Rr+r^2$ is nearly $3R^2$ when thickness is small; surface area is therefore $4\pi R^2$

**page 30**: **1**. –24/25 and –24/7 **2**. cos 311 **3**. circle; ray (or directed line) **4**. **MN** = **NM** only for part **c** **6**. ray makes 63.4-deg angle with pos $x$-axis **7**. $\theta$ = 42.0+180$n$ **8**. 223.0, 317.0, 583.0 deg **9**. $\theta$ = –63.4 **10**. sin 76 **11**. $5^{12}$ ; $\left(b^x\right)^y = b^{xy}$ **12**. $10^{24}$ atoms **13**. $y$ = –2 sin $x$

**page 31**: **1a**. $h(0)$ = 1 is initial height **1b**. $h(12+k) = h(12-k)$ **1c**. $t$ = 6.48 is time needed to reach altitude of 10 meters **1d**. it takes 24 seconds to go once around **1e**. $1 \le h(t) \le 17$ **2**. not a great circle **3**. answer should be 56/169 **4**. $P'$ = (2.870,4.557); matrix method is more efficient **5**. 50.3 m **6**. 788.3 cc **7**. a circle parallel to the Equator through Exeter; a great semicircle (meridian) through Exeter **8a**. the angles $\alpha$ and $\beta$ are in standard position **8b**. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ **8c**. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

**page 32**: **1a**. 2/3 **1bc**. **v**=(2/3)**u** **1d**. they would point in different directions **2**. there are two triangles: $q$ = 15 and $Q$ = 67.4 deg; $q$ = 1.8 and $Q$ = 6.4 deg **3**. $-\sqrt{1-k^2}$ and $-k/\sqrt{1-k^2}$ **4**. (–3,–4,12) **5**. fewer than 1728 **6**. $x$ = –6.92, $y$ = 4 **7a**. components not proportional; **w** = [–57/26,–19/26,76/26] **8a**. $m$ = 19/26 = 0.731; $y = mx$ **8b**. the residual vector **r** = [5/26, –7/26, 2/26] = (1/26)[5, –7, 2], which is perpendicular to [–3,–1,4] **8c**. "squares" refers to the distance formula

**page 33**: **1**. 197060797 sq mi; 19158689 **2a**. $5^{16}$; $b^x / b^y = b^{x-y}$ **2b**. $1.2\times10^2$ or 120 persons **2c**. $0.5\times10^3$ or 500 sec **3**. a thin sector is like a triangle **4**. three-dimensional analogue of preceding **5**. same amplitude, different $x$-intercepts **6**. \$10.50 child, \$18 adult, \$12.50 senior citizen **7**. $G$ = (5, 4.7) lies on $y$ = 0.5$x$+2.2

**page 34**: **1**. 0 (perpendicular vectors) **2**. $b^{1/n} = \sqrt[n]{b}$ **3**. 27/64 = 42.2%; 10.9 cm **4b**. $30m^2 - 16m + 6$ **4c**. $m = \frac{4}{15}$ **5a**. unit vectors, mutually perpendicular **5c**. fixed point; probably the reflection in line $y = 2x$ **5d**. reflections are self-inverse **6**. translations by [0,1] **7a**. $x$-axis reflection **7b**. $y$-axis reflection **7c**. half-turn around the origin

**page 35**: **1**. $h$ = 13.86 **2**. $r$ = 10/3; 40/81 = 49.4% occupied **3a**. $y = 0.2025x - 0.0066$ **3b**. $t^2 = 0.2025h - 0.0066$; $t$ = 7.79 sec **4a**. 28.1 deg **4b**. 85/4 = 21.25 in **5**. 2.99 in **6**. $5^{-4}$ ; reciprocals **7**. division by 10 shifts decimal place; $2.827\times10^{-25}$ sq cm; $1.414\times10^{-38}$ cc **8**. 210 deg and 330 deg **9a**. $(106 - x^2)/90$ **9b**. $(58 - x^2)/42$ **9c**. supp angles; opposite cosines; $x = \sqrt{806/11} = 8.56$ **9d**. $BD = \sqrt{682/13} = 7.24$ **10**. –1; opposite unit vectors

**page 36**: **1**. $x = r\cos\theta$ and $y = r\sin\theta$ **2**. (943,–2738,2701) **3**. burger \$1.89, shake \$0.89, and fries \$0.69 **4**. **MN** is a counterclockwise quarter-turn; **NM** is a clockwise quarter-turn **5**. $h(t) = 7 - 6\cos 24t$ **6**. (5,10) and (20,10) **7**. –53.1, 53.1, 306.9, 413.1 **8**. $12\sqrt{2}$ = 16.97 in **9**. 4.37 in **10**. translations do not have fixed points

**page 37**: **1**. $(b+c)^n$ is not equivalent to $b^n+c^n$, but $(b\cdot c)^n$ is equivalent to $b^n\cdot c^n$ **2**. probably not **3**. 57.3 deg **4**. $2\pi = 6.28$; no **5**. 1.5 radians = 85.9 deg **6**. 15 inches **7a**. $2a^4$ **7b**. $9p^6q^8$ **7c**. $b$ **7d**. $\frac{4}{9}x^6y^{-4}$ **7e**. $d^3$ **8**. 72

**page 38**: **1**. $b^{1997}/b^{1997}$ **2**. $J^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ **3**. shear transformation; image parallelograms **4**. –12/13 and –5/13 **5**. $4\pi r^2$ is 2/3 of $4\pi r^2 + \pi r^2 + \pi r^2$ **6**. 180 degrees = $\pi$ radians **7**. same amplitude, different period **8**. \$1080; \$1166.40; \$1259.71; $\$1000(1.08)^t$ **9**. different directions; different lengths

**page 39**: **1**. 76.4 deg **2**. 76.4 deg **3**. arc length *AB*/radius *OA* **4**. 16.92 **5**. 11/9 is half the size of angle *POQ*, in radians **6**. $\pi/4 = 0.785$; $5\pi/12 = 1.31$; $5\pi/9 = 1.75$; $\pi = 3.14$; $2\pi = 6.28$ **7**. a parallelogram **8**. box it; as in Math 2 **9**. 360; 180; 180; 360/*m* **10**. $2\pi$; $\pi$; $\pi$; $2\pi/m$ **11**. wrong; right; wrong

**page 40**: **1**. $t = 0.4503\sqrt{h} - 0.0033$; $t$ = 7.80 sec **2**. $\sin(-x) = -\sin x$; $\cos(-x) = \cos x$ **3**. $x = 57.0+180n$; all $m$ **4**. $y = \sin x$; $y = -\cos x$; $y = 2\cos x$ **5**. 6693; $5280(1.024)^t$; 29.2 yrs **6**. 5280 = initial population; $t$ = 29.2; $P$ = 2592; **7a**. $x = a^{3/5}$ **7b**. $x = a^{15}$ **7c**. $x = 0.0454$ **7d**. $x = \pm a^{-1/2}$ **8**. 5cm; 11 cm

**page 41**: **1**. a line; a plane **2**. $x$ = 3; $z$ = 4; $y$ = 6 **3**. the left side = initial investment plus two interest payments **4**. $a$ = 7; $k$ = 2 **5**. (1.4, 4.8); (–1.76, 4.68); (5cos 36.87*n*, 5sin 36.87*n*); 3.218 mps; object returns every 9.764 sec **6**. 6859 cc; 6351 cc; 2868 cc; $8000(0.95)^n$ cc; 7797 cc; $8000(0.95)^{k/24}$ cc; 13.5 days **7**. 40 balls fit snugly; 41 balls will fit, but loosely **8**. the axis intercepts of this *plane* are (27,0,0), (0,27/2,0), and (0,0,9); the *yz*-plane intersection is a line; (3,6,4) fits the equation **9**. this perpendicular bisector is the plane $x + 2y + 3z = 27$

**page 42**: **1**. 8000 is initial volume; $t$ = 13.5; $V$ = 9331 is prev volume **2**. (3357,1823,–1045) **3**. $1/E = 0.00068d^2 + 0.00203$; $E$ = 1.13 **4a**. $\frac{5}{2}$ **4b**. $6x^2$ **4c**. $-\frac{8}{27}$ **4d**. $2x^2y^3$ **4e**. $\frac{9}{4x^6}$ **5**. $y = -2\cos x$; $y = 1 + \sin x$; $y = \sin 2x$ **6**. 10.4% **7**. $r = (0.75V/\pi)^{1/3}$

**page 43**: **1**. same *y*-intercept, different slopes **2**. $4000(1.005)^{12n}$ is the same as $4000(1.0617)^n$ **4**. [2,3,5] is perpendicular to any segment lying in the plane; joining (2,2,1) and (0,5,0), for example **5**. proves that [*a*,*b*,*c*] is perpendicular to the plane $ax + by + cz = d$ **6**. $y = -\tan x$ **8**. many possibilities; determinant must be zero **9**. (–1,–4,9) **10**. 51° or 0.89 radians **11**. a *nautical mile* is 1.152 mi = 6082 ft

**page 44**: **2**. put the ends together **3**. (5,0) and (–5,0); (0,4) and (0,–4) **4**. the new table says that $\sqrt{d} = \sqrt[3]{t}$; 4.43 yrs ($d$ = 2.70); 1262 million mi (13.57 au) **5a**. $d = t^{2/3}$ **5b**. $t = d^{3/2}$ **5c**. $d^3 = t^2$ **6**. instead of $d^3t^{-2} = 1$, it would be $d^3t^{-2} = 8.044\times10^{23}$

**page 45:** **1a**. 1 **1b**. $2b^2a^{-1}$ **1c**. 1 **2**. (4, 6, 12) is closest **3**. common asymptote, different *y*-intercepts **4**. $y = 1 + \sin 30x$ in deg mode; $y = 1 + \sin\frac{\pi x}{6}$ in radian mode **5**. a cylinder **6**. a line; $(x, y, z) = (1,1,2) + t[16,1,-7]$ **7**. to graph top and bottom halves separately: $y = \pm\frac{4}{5}\sqrt{25 - x^2}$ **9**. $\sqrt{z^5}$ ; $\sqrt[10]{z^3}$

**page 46**: **1**. 12, 18, 27, 40.5, $8(1.5)^{n-1}$ **2**. what does $(-8)^{1/6}$ mean? **3**. $k = -2$; $a = 5$; $-2.936$; $-1.777$; 0.206; 1.365; 3.347; 4.507; 6.489 **4**. 29.3%; 2.2% **5**. $y = 3^x$; $y = 2\left(\frac{2}{5}\right)^x$; $y = \left(4^{1/3}\right)^x$ **6**. $\cos\beta$

**page 47**: **1**. $4x + 7y - 4z = 9$ **2**. in degree mode, $y = 2 + 4\sin 3x$ will do; in radian mode, $y = 2 + 4\sin\frac{\pi x}{60}$ will do **3**. place *P* so that $F_1$, $F_2$, and *P* are collinear **4**. place *P* so that $F_1$, $F_2$, and *P* form an isosceles triangle; $b = \sqrt{80}$ **5**. always $a^2$ **7**. 183.7, –183.7 **8**. one is $x + 2y = 0$ **9**. 18.2% **10**. $9x^2 + 5y^2 = 45$

**page 48**: **1**. $2\pi$; $(0.5\pi - 1, 1)$; $x = t - \sin(180t/\pi)$, $y = 1 - \cos(180t/\pi)$; or (in radian mode) $x = t - \sin t$ , $y = 1 - \cos t$ **2**. use a rule of exponents **3**. 79.1% per decade; 0.49% per month **4**. $a = 3$, $b = 2$, $c = \sqrt{5}$; one parametric description is $(x,y) = (2\cos t\ ,\ 3\sin t)$ **5**. $10^3$, $10^6$, $10^{-2}$, $10^{1/2}$, $10^{5/2}$, $10^{-2/3}$ **6**. $1997 = 10^{3.300}$; $1997^{-1} = 10^{-3.300}$ **7**. 3.300 **8**. 3.000; 6.000; –2.000; 0.500; 2.500; –0.667 **9**. $x = 4020$

**page 49**: **1a**. wrong **1b**. wrong **2**. 0.477; 2.477; 0.954; $-0.477$; 0.239 **3**. $25x^2 + 81y^2 = 2025$ **4**. $2^{16} = 65536$ **5**. $(x, y) = (m\cos t, n\sin t)$ **6**. $d = 0$; $d = 9.75$, $d = 30.00$, $d = 2865$; $d = 30$ tan 60*t* is not linear **7a**. $10^4 = 10000$ **7b**. 3.30 = log 1997 **8**. $p(0)$=3960; 2% per year; 1.02 **9**. $t = 150.00$

**page 50**: **1**. $a = 3$, $b = 2$, $c = \sqrt{5}$; $a = 7.5$, $b = 5$, $c = 2.5\sqrt{5}$; eccentricities both $\frac{1}{3}\sqrt{5}$; yes **2**. $0 \le$ ellipse ecc $< 1$; circle ecc = 0; ecc increases **3**. major is longer; false **4a**. traced clockwise; starts from (1,5) **4b**. traced counterclockwise, starts from (1,5) **4c**. traced counterclockwise; starts at (4,2); twice as fast **5a**. $9x^2 + 25y^2 = 900$ **5b**. $25x^2 + 9y^2 = 2025$ **6**. all three answers are 5/3 **7**. $64 = 4^3$; $12 = 3 \cdot 4$; $0.02 = 1/50$ **8**. $t = 1.44$ and $t = 8.4085$ **9**. $7x + 4y + 4z = 33$ **10**. (–0.4,9.2) and (10.4,–5.2)

**page 51**: **1a**. $k = 3$, $b = 4$ **1b**. $k = 4$, $b = 0.5$ **2**. axis intercepts at (0,–2), (0, 6), (–3, 0), and (3,0); $y + 6 = 2\sqrt{x^2 + y^2}$ **3**. $a = 4$, $b = \sqrt{12}$, $c = 2$ **4**. the same curve as the preceding **5**. $x = 55.48$ **7**. 76.3%, compounding daily interest **8**. $a = 15$, $c = 9$ **9**. $G = (-9,0)$; directrix is $x = -25$; $PF + PG = 30$

**page 52**: **1**. $\frac{1}{2}\sqrt{30} = 2.74$ **2**. slide 90 in the positive $x$-direction **3**. 90.7% **4**. on 31st day, second option pays $2^{30} = 1073741824$ pennies, which is more than \$10 million **5**. penny thickness = 1/17 inch, diameter 0.75 in, volume = 0.0260 cu in; vol of $2^{30}$ pennies is more than 16148 cubic feet, which is actually the volume of more than eleven dormitory rooms **6**. $2^{9.966} = 1000$ **7a**. 0.774 **7b**. 1.292 **7c**. 0.5 **7d**. 183.7 **7e**. –2 **8**. $\sqrt{x^2 + y^2} = (2/3)(5 - x)$; (–8,0) is the other focus and $x = -13$ is its directrix; $x = -4$ is a symmetry line **9**. $k = 3$, $b = 2$ **10**. yes; 100% growth rate

**page 53:** **1a**. 16 fps **1b**. 24 fps **1c**. 30 fps and 34 fps **1d**. 32 fps **1e**. 58 fps **1f**. 68 or 70 fps **2a** 0.778 **2b** 0.903 **2c** –0.176 **2d** 0 **3a** 0.778 **3b** 0.903 **3c** –0.176 **3d** 0 **4**. slide the sine graph 90 to the left **5a**. $a = 10^m$, $b = 10^n$, $ab = 10^k$ **5b**. $m + n = k$ **7**. 31416; 20432; 24% per day; 0.9% per hour; $G(x) = 31416(1.009)^x$ or $G(x) = 31416(1.24)^{x/24}$ **8**. $(x, y, z) = (2,-3,5) + t[8,-10,11]$

**page 54**: **1**. the vertex is closest to the directrix, so… **2**. in degree mode, $y = 1 - 3\cos 9x$ fits the data; in radian mode, $y = 1 - 3\cos\frac{\pi x}{20}$ fits; there are other answers as well **3**. 1.861 **4**. (log $N$)/(log $a$) **5**. it is wrong **6**. (0,1,4) and (4,5,0) are two of the points; the line is $(x,y,z) = (0,1,4) + t[1,1,-1]$ **7**. $(1.01)^{12} = 1.1268$, so growth is 12.7% per year **8**. 11.335 cm; 0.623 cm/sec (0.626 cm/sec instantaneous)

**page 55**: **1**. equilateral triangle **2a**. $a = 20$; $b = 12$; $c = 16$; ecc = 4/5 **2b**. $P = (12\cos t, 20\sin t)$; $F = (0,16)$ **2c**. $y = 25$ **3**. $z$-intercept and two slopes **5**. $\log y = 3 + 2\log x$ **6**. $M(x) = 14.7(0.8)^x$; $F(x) = 14.7(0.8)^{x/5280}$; 4.29 lbs/sq in; 15.53 lbs/sq in **7**. $f(x) = 5 + 3\sin 30x$ is one of many examples **8b**. 2.226 sec **8c**. 1.228 cm/sec (1.243 cm/sec instantaneous) **8d**. 0.309 cm/sec (0.311 cm/sec instantaneous)

**page 56**: **1**. $t = 50.19 + 180n$; $P = (1.92,3.84)$; $r = 4.29$; $\theta = 63.43$ **2**. [11,7,–5] is one **3**. \$20084.52 **4**. $\log t = (1.5)\log d$ **5a**. $\log y = 3\log x$ **5b**. $\log y = 2 + 0.5\log x$ **5c**. $\log x + \log y = 3.598$ **5d**. $(2/3)\log y = 0.699 + (5/2)\log x$ **6**. $x^{5/2}$; $x^{1/2}$; $x^{1/6}$; $xy^{4/3}z^{5/3}$ **7**. error; negative numbers do not have logarithms **8**. $F(t) = 3960(4/3)^t$, so $F(-17)$=29.8 ants

**page 57**: **1**. $y = 3x^4$ **2**. $x\sqrt{y} = 300$; $xy^{3/2} = 500$ **3**. $81(2/3)^{39} = 0.000011$; $81(2/3)^{999999}$; $81(2/3)^{n-1}$ **4b**. $3 \cdot 4^n$ **4c**. $3 \cdot 4^n$ **4d**. 1/3; 1/9; $(1/3)^n$ **4e**. $3(4/3)^n$ **4f**. no **4g**. yes; it is contained inside a hexagon **5**. 4.03; –0.51; 0.757; 1.76$m$; –3.52 **6**. $2x + 3y + 6z = 5$

**page 58**: **1**. $(x,y,z) = (1,1,1) + t[11,-5,7]$ **2**. $\cos^{-1}(1/14) = 85.9$ deg **3a**. 1/8 **3b**. $8^{1/16} = 1.139$ **3c**. 7/2 **4**. focal points $(\sqrt{12}, 0)$ and $(-\sqrt{12}, 0)$; eccent = 86.6% **5**. another is 9, 12, 16; multiplier must be between 0.618 and 1.618 **6a**. $p(t) = 30\tan 60t$ **6b**. 17.32 ft **6c**. 34.64 fps **6d**. 0.314 ft **6e**. 31.4 fps **7**. 7.702691, 8.038476, 8.041843, 8.0721567, 8.376151 **7a**. 33.67298 fps **7b**. 33.68095 **7c**. 33.67243 fps **7c**. third is best (true speed 33.67149 fps); see commentary **8**. four **9**. Eugene is mistaken; *C* does not fit the equation **10**. $4x - 27y + 4z = 296$ **11**. 1280; 14580

**page 59**: **1**. 13.96 fps; 12.09 fps; $y = 20\sin(40t)$ describes position of shadow **2**. the shadow is 10 feet from *N*; *Q* = (0.751, 10.01209); the slope = 12.09 = speed of shadow **3**. $2^{216091} = 74609\cdots 8$ is a 65050-digit number **4**. 72 million km **5**. LinReg gives either $\log(P) = 0.5217\log(L) - 0.7373$ or $\log(L) = 1.9011\log(P) + 1.4158$; thus either $P = 0.1831L^{0.5217}$ or $L = 26.049P^{1.9011}$; $P = 7.5$ sec corresponds to either $L = 1232$ cm or $L = 1201$ cm **7**. 34, 55, 89, ... **8a**. 4.723 **8b**. 220.3 **8c**. –2 **8d**. 4.723

**page 60**: **1**. $2^{39}$ and $3^{17}$ **2**. 55 rabbits on 1 June; no; no **3**. (0.01, 55.01); 1.09 rabbits per day; 6.74 rabbits per day **4**. 79.4% as deep as the cup **6**. both angles are 63.44 degrees **7**. $x = 3\cos t$ and $y = 2\sin t$ for $0 \le t \le \pi$; $y = (2/3)\sqrt{9 - x^2}$ **8**. the area of a $3\sqrt{2} \times 2\sqrt{2}$ (4.24 by 2.83) rectangle is 12 **9**. $x + 4y + 8z = 81$ **10**. 176091 zeros **11**. the $n^{\text{th}}$ number is *n* more than its predecessor; not geometric

**page 61**: **1**. the mean distance is *a* = 17.94 a.u.; the eccentricity is 17.35/17.94 = 96.7%, and the greatest distance is 35.29 a.u. = 3292 million miles **2**. slope at (0,1) is approximately 0.693 **4**. reflect in line $y = x$ **5**. –2.48; 4.8 **7**. $H = 30k^{-5/2}$ **8a**. $2x + 3y = 4$ **8b**. $2x + 3\log y = 4$ **8c**. $2\log x + 3y = 4$ **8d**. $2\sqrt{x} + 3y = 4$ **9**. if $k\log(x) + m\log(y) = n$, then $x^k y^m = 10^n$ **10.** constant exponent vs. variable exponent **11**. slide first graph by vector [0,1] **12**. $y^{2.54} = 5495x^{3.14}$ **13**. focus is (0,1/4); directrix is $y = -1/4$

**page 62**: **1**. (16,0) and (−16,0) are the vertices; also $(0,\sqrt{112})$ and $(0,-\sqrt{112})$ **2**. the two rows that contain 0 must be deleted **3**. 40; 33.3; 7.75 **4**. answers will vary the (the roll most likely to end the game is the 22nd) **5**. reflect one graph in the line $y = x$ to get the other graph **6**. $9x^2 + 25y^2 = 225$ and $3x^2 + 4y^2 = 192$ both have their focal points at (4, 0) and (−4,0); yes, confocal implies concentric **7**. slide by vector [3,–1] **8**. slide by vector [3,–1] **9**. $x = 4$; $x = 3$; $3 < x$; $x = 8$; $x = 28$; $x = 128$ **10**. slide by vector [3,0]; slide by vector [5,0]

**page 63**: **1**. $x = 4$ and $x = -4$ **2a**. –69.59 = slope (approx) at (2.00,1704.96) **2b**. $x = 26.58$ **3**. 250.3 **4**. $y = 2^{x-3}$ has no $x$-intercept, $y$-intercept 0.125, and is asymptotic to $y = 0$; $y+5 = 2^x$ has $x$-intercept 2.322, $y$-intercept –4, and is asymptotic to $y = -5$ **5**. $y = -0.875$; slide by vector [3,–1] **6**. slide by vector [3,–1] **7**. both angles are 60.3 deg **8**. left pile has $9 \cdot 10^9$ stones and the right has $10^9$ **9**. $L_n = (1/\sqrt{2})^{n-1}$; $L_{20} = 0.00138$ **10**. $A_n = (1/2)^{n-1}$ is also geometric **11**. $p$ is the distance from the focus to the vertex

**page 64**: **1**. $x = 13$; slide by vector [3,–1] **2**. $10^{1997} = 2^{6633.89}$ **3a**. 12/7 **3b**. $3^{2000}$ **3c**. $7^{2u}$ **3d**. $8x^8$ **3e**. $(2/3)^m$ **4**. $\log_{16} N = (\log N)/(\log 16)$ **5a**. $f(x) = 8\cos 6x$ **5b**. $g(x) = 8\sin 6x$ **6**. 8.553, 51.45, 68.553 **7**. (9, 0, 8) is the point of tangency **8**. 448/3; 189/4 **9**. $x$ is between 0.25 and 2 **10**. 26.39 ft; length is 9 csc $t$ + 8 sec $t$, which is smallest (24.03 ft) when $t$ = 46.12 deg **11**. $\frac{1}{6}$; $\frac{5}{6}$; $\left(\frac{1}{6}\right)^3$; $\left(\frac{5}{6}\right)^3$ **12a**. 6 **12b**. 4 **12c**. 6 **13**. $y = a\left(x+\frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$ has its vertex at $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$ and its focus at $\left(-\frac{b}{2a}, \frac{1+4ac-b^2}{4a}\right)$; the line $y = \frac{4ac-b^2-1}{4a}$ is the directrix

**page 65**: **1**. each logarithm is 0.602 more than its predecessor **2**. G = 392.6 cps; $262 \cdot 2^{n/12}$ **3**. the ratios are all $2^{7/12} = 1.498$ **4**. 9/20 **5**. $t$ = 126.8699 puts a point very close to $P$ **6**. (8, 3) **7**. $A_n = A_0(8/9)^n$; $A_{40}$ is less than 1% of original area **8a**. 8/9 **8b**. $(8/9)^2$ **8c**. $(8/9)^n$

**page 66**: **1**. $(5/6)^4(1/6) = 8.0\%$ **2**. 1599840; 1599680; 1599520; $1600000(0.9999)^d$ **3**. $n$ = 2001; the multiplier is 1.027; $U_{1950+k} = 25791017(1.027)^k$ **4**. 53309655 miles **5**. 8.0%; 71.6%; 5.7% **6**. 8.067 is approximately the slope of the line tangent to $y = 5^x$ at (1,5) **7**. reflect across the $y$-axis; opposite slopes at (0,1) **8a**. on 20 July, $R$(48.85) = 144.7 = $B$(48.85); 2.87 rabbits/day; 1.44 beavers/day **8b**. on 28 September **9**. $3x + 2y + 6z = 36$ **10**. 112 and 68

**page 67:** **1**. area = $15\pi$ **2**. $\pi ab$ **3**. $m \approx 1.099$ **4.** $m \approx 1000(b^{0.001} - 1)$ **5**. the slopes seem to follow a logarithmic pattern **6**. 2; –1; 1.236; slide given graph by the vector [3,–1] **7**. 20.900 **8**. 26361 miles **9**. each satellite can see 45% of the equator **10**. $V_n = 0.88V_{n-1}; V_0 = 5000$

**page 68**: **1**. circle tangent is $-3x + 4y = 25$; ellipse tangent is $-9x + 20y = 75$ **2**. the reflection principle **3**. the angle bisectors are perpendicular **5a**. 1.68 m/s **5b**. 2.09 m/s when height is 6 meters **6**. $y$ = 7.03; [$x$,$y$] = [–366,443]; $y = \pm(1/3)\sqrt{144 - 16m^2}$ **7**. 421671 km (Io); 7.151 da (Ganymede); 16.65 da (Callisto) **8**. 4 cents per dollar per year **9**. $2x+3y = 27$ **10**. using degree mode, the average speed = 8; greatest speed = 12.56

**page 69**: **1a**. 460 in Exeter **1b**. 538 in Exeter **2a**. how to calculate the expected Tuesday am distribution from the Monday am distribution **2b**. $\mathbf{M} = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$ **3a**. expected distribution on Wed morning **3b**. distribution a week from Thursday **3c**. upper right corner is the probability that an Exeter car finishes its day in Hampton **3d**. no cars enter or leave the system **4**. $x$ = 4000/7 = 571 **5a**. P is equidistant from $F$ and $N$ **5b**. except for $P$, all points on the parabola are closer to $F$ than they are to $N$ **5c**. triangle $FPN$ is isosceles **5d**. angle between $\lambda$ and $\mu$ equals angle between $FP$ and $\lambda$ **6**. in all cases, $p$ is between 1 and $b$

**page 70**: **1**. 999879; 5728 yrs **2**. 9416 yrs **3**. 1717 ft **4**. 8.0%; 71.6%; 5.7% **5c**. 1.2 cm; no; 188.5 cm/sec

**page 71:** **1a**. 2204365381900 kwh; 2516398838 bulbs **1b**. 31.1 times **1c**. $E = 10^{(R+1.17)/0.67}$ **1d**. 31.1 **1e**. yes; $R < 0$ indicates a very small seismic wave; a 55.8 kwh wave is rated 0 **2**. \$3207.14 **3**. \$3025.60, \$2854.34, \$2692.77, … $\$1000(1.06)^k$ **4**. \$39992.73 **5**. the sum of a geometric series is (*first – last*multiplier*)/(1 – *multiplier*)

**page 72:** **1.** $3383\left((1.04)^{n+1}-1\right)/0.04$ **2a**. $yx^{2.3} = 70.0$ **2b**. $y = 1600.0(0.309)^x$ **3**. $3x+6y+2z=23$ **4**. $y = 2x-16$ **5**. at the *Loaf*: 46.5%; 45.2%; 44.4% **6**. $2^x$; $4.36(1.4)^x$; $\log_m k$ **7.** 202.5 **8**. 6.75 ft; 0.12 ft; 63.52 ft **9**. 47.64 ft; 111.16 ft **10**. \$119179

**page 73**: **1**. 1.76% per year, or 93814669 million persons per year (yearly averages); 1.745%, or 92998659 persons per year (instantaneous rates) **2**. 1.76% per year, or 111697056 million persons per year (yearly averages); 1.745%, or 110725505 persons per year (instantaneous rates); yes **3.** *x min* = –21, *x max* = 63, *y min* = –4477, *y max* = 13431 will do **4c**. 0 cm **4d**. $x$ = 45 cm; $x$ = 135; 251.3 cm/sec **5**. (0.5,30) **6**. inverse sine values only between –90 and 90, inclusive

**page 74**: **1**. log(5) – log(3) ≠ log(2), because log(5) – log(3) = log(5/3) **2**. 80; 83; 70 **3**. $1-r^2$; $1-r^3$; $1-r^{1996}$ **4**. 107157.76; 10.8 **5**. 29.82; 110; $\sum_{n=1}^{19} n^2$; $\sum_{n=-3}^{3} 2^n$ **6a**. 64.0 feet **6b**. 112.0 feet **6c**. 112.0 feet **7**. parenthesis fatigue; makes $\sin^{-1}x$ look like a reciprocal; there is the dilemma of finding a meaning for $\sin^{-2} x$

**page 75**: **1a**. 4, 2.828; 2; 2.828 **1b**. $2 \le f(t) \le 4$ **2**. $f(t) = \cos(90t) + \sqrt{9-(\sin 90t)^2}$ **2a**. 0.8934 and 3.1066 **2b**. notice that g(1) = 3 **2c**. no **3**. –1.5708 ups; from cubic formula, max velocity is –1.6566, when $t = \sin^{-1}\left(\sqrt{3-12\sin 10}\right)/90 \approx 0.8131$ **4**. (1.0,45.0) **5**. 12 **6**. the *associative* property is **M**(**NP**) = (**MN**)**P** **7**. $\cos x$ **8**. $\sin(x+0.5\pi) \equiv \cos x$

**page 76**: **1a**. \$46296.30 **1b**. \$36751.49 **1c**. \$362344.40 **2**. there is one place where lightning could have struck the road **3**. –1 to 1, inclusive; all nonnegative numbers; all positive numbers; all numbers **4**. 0 to 180, inclusive; all nonnegative numbers; all numbers; –5 to 9, inclusive; depends on the sign of the coefficient: all positive numbers *or* all negative numbers **5a**. 0 **5b**. $p \cdot (1-m^{w+1}) / (1-m)$ **6**. all inverse tangent values are between –90 and 90 **7**. $y = \sqrt{mn}$ **8**. in degree mode, $a = 3$, $m = 2$, $b = 90$ **9**. 80%; 71.5%; 65.2%

**page 77**: **1**. Kim could also have figured out that the lightning must have struck either 3 miles north or three miles south of Pat **2**. the expression $LK - LP = 2$ is positive on Pat's side of town **3**. 3.0; 6.0; 11.9; 23.7; as $S_n$ gets larger, the percentage $1 - P_n$ of persons who already know the rumor becomes large enough to inhibit further increases in $S_n$ **4a**. $P_{10} = 0.954$ **4c**. between 8 pm and 9 pm **4d**. $P_{n+1} = P_n + 2P_n \cdot (1 - P_n)$ **5**. the sum is 1 **6**. $3x^2 - y^2 = 3$

**page 78**: **1**. 180 or $\pi$; sinusoidal graphs do not have corners **2**. half-life; half-life **3**. use the $10^x$ function **4**. 13/99 **5**. $T = \sqrt{3}$, so $x = 60 + 180n$ **6a** $P_1 = 1/4; U_1 = 3/4$; $P_2 = 3/16; U_2 = 9/16$ $P_3 = 9/64; U_3 = 27/64$ **6b**. $U_n = (3/4)U_{n-1}$; $U_n = (3/4)^n$ **6c**. $P_n = (3/4)P_{n-1}$; $P_n = (1/4)(3/4)^{n-1}$ **6d**. $P_1 + P_2 + P_3 + \cdots P_{100} = 1.0000000$ **6e**. $\sum_{n=1}^{101} P_n = \sum_{n=1}^{101} 3^{n-1} / 4^n$ **7**. \$94.83 million **8**. the period is 360 degrees or $2\pi$ radians **9**. 12 is a period (the least common multiple of 4 and 6)

**page 79**: **1**. in large windows, the graph resembles the two intersecting straight lines **2**. the asymptotes are $y = \pm x\sqrt{3}$ **3**. constant differences **4**. $3x^2 - y^2 = 3$ **5**. $\text{temp} = 20 + 70(0.8)^{t/5}$ **6**. in degree mode, the period is 180; it is sinusoidal; range is $|y| \le \frac{1}{2}$ **7a**. 0.32 **7b**. 1.61 **7c**. 123 **7d**. 43 **8**. $f(x) = 5 + 4|\sin 18x|$ , $f(x) = 7 + \frac{1}{45}\sin^{-1}(\sin 36x)$ , ...

**page 80**: **1**. *something* has to happen **2**. 1 **3**. hyperbola is $9y^2 - 16x^2 = 144$; asymptotes are $9y^2 - 16x^2 = 0$; $(0, \pm 4)$; $(\pm 9/4, \pm 5)$ **5**. identical **7**. **8**. $A = 5$; $p = \sin^{-1} 0.8 \approx 53.1$ **9**. the answer (0) is correct, but Kirby made two egregious errors **10**. look at an angle bisector in a 13-10-13 triangle **11**. 6; 4; 12

**page 81**: **1**. half-life about 6.0 **2**. 1/365; 364/365; (364/365)(364/365); $(364/365)^{1017}$; $1 - (364/365)^{1017} = 93.9\%$; 6.1% **3**. it is negligibly small; it is nearly equal to $a/(1 - r)$; 12 + 4 + 4/3 + 4/9 + ... **4**. $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ and $0 \le \cos^{-1} x \le \pi$ **5**. $\log(y-8) = x\log 2 + \log 5$

**page 82**: **1**. the total enclosed area is $8/5 = 1 + \{3/9 + 12/81 + 48/729 + ...\}$; the braces contain a geometric series **2**. 3.85 **3**. $-1 < t < 1$; sum $= 1/(1 - t)$; a number that the subtotals get very close to **4a**. 3/7 **4b**. 3/7 **4c**. 1/7 **5**. place $P$ so that $F_1$, $F_2$, and $P$ are collinear

**page 83**: **1**. there are 3 possible dots on the line $x + y = 2$; the dot (1,1) is twice as likely as either (2,0) or (0,2) **2**. there are four dots on the line $x + y = 3$; the dots (2, 1) and (1, 2) are each three times as likely as either of (3,0) or (0,3) **3**. each of *rru*, *rur*, and *urr* describes a different path to (2,1) **4**. calculate $(r+u)^5 = r^5 + 5r^4u + 10r^3u^2 + 10r^2u^3 + 5ru^4 + u^5$; there are 10 paths to (3,2); probability the bug will land there is 10/32=31.25% **5**. 62.8; 2.8 + 1.2million; 2.8 + 1.2*n* **6**. 5 **7**. $0.138918(1+10^{-6}+10^{-12}+\cdots) = 138918/999999 = 3562/25641$ **8**. $y = \sqrt{5}\sin(x - \tan^{-1}(1/2))$ **9a**. vertices $(0,\pm 2)$; foci $\left(0,\pm\sqrt{13}\right)$; asymptotes $3y = \pm 2x$ **9b**. vertices $(\pm 3,0)$; foci $\left(\pm\sqrt{13},0\right)$; asymptotes $3y = \pm 2x$ **10**. each column is a probability vector **11**. multiplier is 1/5; limit is (10.5,3.5) **12**. 12.8csc25 **13** the sine is $\sqrt{1-k^2}$, the cosecant is $1/\sqrt{1-k^2}$, the tangent is $\sqrt{1-k^2}/k$

**page 84**: **1**. 1/7 **2**. 15; 15/64 = 23.4% **3**. the *n*th row of this array contains the coefficients of the expanded binomial $(x+y)^n$ **4**. 254/495 **5a**. $4x = 3y$, $4x = -3y$ **5b**. $y = -3$ **5c**. $x = -3$ **6**. 1/3 **7**. 327 feet and 3 inches **8a**. **M** is a constant multiplier **8b**. vector tips lie on $x + y + z = 100$ **8c**. the terms of the sequence approach a *limit* vector

**page 85**: **1**. a family of parallel lines **2**. ten **3**. 5050 **4**. 87156; $n(n+1)/2$ **5**. (0,4), (0,–4); $y = \pm 2x/\sqrt{5}$ **6**. (0,9) and (0,–9) are the vertices; also $(\sqrt{45},0)$ and $(-\sqrt{45},0)$ **7**. slope = –3/2 **8**. $3y = 2x + 8$ is tangent to $5y^2 - 4x^2 = 80$, and it bisects the angle formed by the focal radii **9**. curve can be described by $y = h + 1 - \cos\theta$ **10**. 299; 1336; 1958; populations decrease toward 2000

**page 86**: **1**. $y = \sqrt{10}\sin(x + \tan^{-1}(3))$ **2a**. 0.1615 **2b**. 0.8385 **2c**. 0.0323 **2d**. 0.0323; **2e**. 0.3230 **3**. divide both sides by $(\cos t)^2$ **4**. the hyperbola $25x^2 - 9y^2 = 225$ has asymptotes $25x^2 - 9y^2 = 0$ **5**. only the asymptotes are visible from a great distance; a parabola looks like a ray **6**. 504 **7**. 84 **8**. 3/84 ; 0 **9**. $y = \pm(2/3)\sqrt{x^2 - 36}$ or $(x, y) = (6\sec t, 4\tan t)$ **10**. $(b^h - 1)/h$; note that *a* and *t* are missing from the answer; annual relative population growth rate (persons per person per year) does not depend on the size of the population **11**. annual population growth rate, in relation to actual population, is constant **12**. $f(x) = 2 + \frac{1}{18}\sin^{-1}(\sin 7.2x)$

**page 87**: **1a**. $4x^2 - 9y^2 = 144$ **1b**. $4x^2 - 9y^2 = -225$ **2**. slope 2; yes, asymptotes of a hyperbola **3a**. $-3 \le x \le 3$ and $0 \le y \le 6$ **3b**. $-1.5 \le x \le 1.5$ and $0 \le y \le 3$ **3c**. $-3 \le x \le 3$ and $2 \le y \le 5$ **3d**. $-5 \le x \le 1$ and $0 \le y \le 3$ **4**. $x = -8$ ( prob 1/256); $x = -6$ ( prob 8/256); $x = -4$ (prob 28/256); $x = -2$ (prob 56/256); $x = 0$ (prob 70/256); $x = 2$ (prob 56/256); $x = 4$ (prob 28/256); $x = 6$ (prob 8/256); $x = 8$ (prob 1/256); total probability = 256/256 **5**. $x = \pm 9$ (prob 1/512 each); $x = \pm 7$ (prob 9/512 each); $x = \pm 5$ (prob 36/512 each); $x = \pm 3$ (prob 84/512 each); $x = \pm 1$ (prob 126/512 each) **6**. $8y^2 - x^2 = 72$; $a = 3$; $c = 9$; $c / a = 3$ **7**. 1; 5; 14; 30; 55 **8**. $a$ = 1/3, $b$ = 1/2, $c$ = 1/6, $d$ = 0; volume ratio is $(2n^2 + 3n + 1) / 6n^2$; 1/3**page 88**: **1**. $\sum_{n=0}^{15} 28(0.6)^n = 69.98$ **2**. range dilated; domain dilated; range translated; domain translated **3**. $x = \csc t$ , $y = \cot t$ **4**. $t$ = 1/800; $t$ = 3/800; $x$ = 45; the speed is 1257 cm/sec **5**. 5040 **6**. $y$ = 7sin(1.8$x$) **7**. $y$ = –7cos(3.6$x$) or $y$ = –7cos(10.8$x$) **8**. the center is at (1,2); $16(y-2)^2 - 9(x-1)^2 = 144$; asymptotes are 4($y$ – 2) = ±3($x$ – 1) **9**. 10 **10**. –80 **11**. $(1/6)^2(5/6)^3 = 1.61\%$; 16.1% **12**. $P^{-1}(x) = \log(x / 3960) / \log(1.06)$; 4.937

**page 89**: **1a**. 57 = 25 + 16 + 16 **1b**. 3.25 = 1.25 + 1 + 1 sec **1c**. both have applied the formula carelessly **2**. 11.25 sec; 113.9 ft; 1025/9 feet in 45/4 sec **3a**. $-3 \le x \le 3$ and $-16 \le y \le 0$ **3b**. $-6 \le x \le 6$ and $0 \le y \le 4$ **3c**. $-3 \le x \le 3$ and $-1 \le y \le 3$ **3d**. $0 \le x \le 6$ and $0 \le y \le 4$ **4**. all have 6 as a period; ranges differ: 0 to log 3; 1/2 to 2; 0 to 5 (all inclusive) **5**. 362880 **6**. $1000! = 402387 \cdots 00$ is a 2568-digit number; in how many ways can 1000 Exonians arrange themselves in line while waiting to eat at Elm St Dining Hall **7**. $(365/365)(364/365)(363/365)\cdots(334/365)$ = 0.247; 75.3% is probability of coincidence **8a**. 3.1% **8b**. 13.2% **8c**. 9.3% **9**. $C(x)$ = (5/9)($x$ – 32) **10**. $y$ = log($x$ + 3)/log(2)

**page 90**: **1**. total distance traveled is $h \cdot (1+r) / (1-r)$; time = $(1/4)\sqrt{h} \cdot (1+\sqrt{r}) / (1-\sqrt{r})$ **2**. 3024 **3**. 8; 96; 384; 512 **4**. $(2+8)^3 = 2^3 + 3 \cdot 2^2 \cdot 8 + 3 \cdot 2 \cdot 8^2 + 8^3$ **5**. (2,0) 36%; (1,1) 48%; (0,2) 16% **6**. 34.56% **8c**. (2.626, 0.620) and (–2.626, –0.620) **8d**. ($\sqrt{18}$ , $\sqrt{18}$ ) and ($-\sqrt{18}$ ,$-\sqrt{18}$ ) **9**. focal points (3, 0) and (–3, 0); intersections $(\pm 3, \pm 3.2)$; angle 90 degrees **10**. 50; 80; 400 **11**. domain $-8 \le x \le 12$ and range $-9 \le y \le 15$

**page 91** **1**. second entry is 36 = (9)(8)/(2)(1) **2**. each quadrilateral is associated with a "path" *nnynynnyy*, where "*n*" means "no" (the vertex is not used for the quadrilateral) and "*y*" means "yes" (the vertex is used) **3a**. \$9598; \$9393.98; \$9187.92; \$8979.80 **3b**. $A_n = (1+r)A_{n-1} - P$ **3c**. eventually the recursion $A_n = (1+r)^n A_0 - [P + (1+r)P + (1+r)^2 P + \cdots + (1+r)^{n-1} P]$ appears **3d**. $P$ = 263.34 **4**. the graphs splice together; both are sections of $x$ = sin $y$

**page 92**: **1**. $(5/6)^4$ = 48.2% is prob of no aces when four dice are rolled; $4(5/6)^3(1/6)$ = 38.6% is prob of one ace; $6(5/6)^2(1/6)^2$ = 11.6% is prob of two aces; $4(5/6)(1/6)^3$ = 1.5% = prob of three aces; $(1/6)^4$ = 0.1% is the prob of four aces **2**. geometric sequence; multiplier 2cos 72 = 0.618 **3**. the asymptotes are $x = 0$ and $y = 0$; vertices (4, 4) and (–4, –4); focal points $(4\sqrt{2}, 4\sqrt{2})$ and $(-4\sqrt{2}, -4\sqrt{2})$; eccentricity = $\sqrt{2}$ **4**. 120; 120 **5**. domain $f$: positives only; domain $g$: nonzeros; both ranges all numbers **6a**. log(96!) **6b**. $10+3\sqrt{10}$ **7**. 9 choices for the first letter, 8 choices for the second letter, et cetera **8**. $(x,y) = (3 + 4\sec t, -2 + 5\tan t)$ **9**. $(n-1)!$ **10**. geometric sequence **11**. same ratio as the $x$-sequence **12**. the domain is all $x$; the range is $-90 < y < 90$ (degrees) or $-\pi/2 < y < \pi/2$ (radians) **13**. 288, 387, 325

**page 93**: **1**. $1+4+9+\cdots+14^2 = 1015$; 61.2 in **2**. $1/4 < P < 8$ **3**. $y = 10^b x^m$ **4b**. $A_{48} = -2242.52$ **4c** $P$ = 263.34; $A_n$ depends linearly on $P$ **5**. ${}_{20}C_4 = 4845$, ${}_{19}C_3 = 969$; ${}_{19}C_4 = 3876$ **6**. a series is the sum of a sequence **7**. ${}_9P_4 = 3024$ **8**. $x - y = 0$ and $x + 7y = 0$; the major axis is $x = 3y$; the vertices are (3,1) and (–3,–1) **9**. 256

**page 94**: **1**. ${}_nC_r = n!/(r!(n-r)!)$ **2a**. 380 **2b**. 5/3 **2c**. 5/3 **2d**. $1+r^{20}$ **3**. the 8th row of Pascal's triangle is 1 8 28 56 70 56 28 8 1 **4**. apply eight successive square roots to 5 **5**. $E(90)$, $E(180)$, and $E(270)$ **6**. with a dozen students, ${}_{365}P_{12}/(365)^{12}$ = 83.3%; complementary event is 16.7% likely **7**. with 23 persons, the probability of coincidence is 50.7% **8**. 7.96% **9**. 8/24 **10**. a regular octahedron, which has half of the given volume **11**. $r$ **12**. $649.73

**page 95**: **2**. 495; 792 **3**. twelve payments; $1046.04; $45.41 **4**. 36.7%; 23.5% **5**. $-1 \le x \le 1$; $0 \le y \le 1$; the upper half of the unit circle **6**. $A_n = 1000n + A_{n-1}$ and $A_0 = 0$; $A_n = 500n(n+1)$; $G_n = (0.01)2^{n-1} + G_{n-1}$ and $G_0 = 0$; $G_n = 0.01\left(2^n - 1\right)$ **7**. reflect $D$ across the line to $D'$ and apply triangle inequality to triangle $D'CP$ **8**. apply the preceding

**page 96**: **1**. (–1.8955,–0.9477); (0,0); (1.8955,0.9477) **2**. 50% **3**. $3x - 2y = 8$ **4**. approximately $5.328 \times 10^{17}$ years **5a**. $2011.56; $724161.60 **5b**. $2535.67; $456420.60 **6**. 155 payments (last one $13.12); $3093.12 **7**. parallel planes; the middle one is 20/7 from its neighbors **8**. 6.12% **9**. period 360, range $0 \le f(x) \le 180$ **10**. 1 **11**. in radian mode, the window $[-3.2 < x < 3.2, -1.6 < y < 1.6]$ shows all the details of *both* curves **12**. $y = \cos^{-1} x$

**page 97**: **1a**. $20996011(\log_{10} 2) = 6320429.10$ **1b**. 4214 characters per page is possible **1c**. there is no largest prime **2a**. $9X^2 + 25Y^2 = 225$ **2b**. 10 by 6 **2c**. $\cos^{-1} 0.6$ **2d**. slope is 4/3; vertices (3,4) and $(-3,-4)$ **3**. replace $x$ by $(\cos 22.5)X - (\sin 22.5)Y$ and $y$ by $(\sin 22.5)X + (\cos 22.5)Y$; resulting equation is $\left(1+5\sqrt{2}\right)X^2 + \left(1-5\sqrt{2}\right)Y^2 = 24$ **4**. because a meridian is not actually a circle, one minute of arc varied between 1849 meters and 1855 meters; value standardized in 1929 to be exactly 1852 meters **5a**. \$76190.48 **5b**. \$77097.51; \$65955.66 **5c**. 5.50 years; \$78375.93 is peak present value **6**. $546 / 2187 \approx 0.24966$