Exploring Transformations

You will do these exercises using the Desmos graphing calculator which you can find online at https://www.desmos.com/calculator.

Start by creating function $f(x) = x^2$, and another function, $g(x) = a\,f(b(x - c)) + d$. Desmos will prompt you to create four sliders labeled a, b, c, and d. Set the slider range for a and b to 0 to 10. Leave the sliders c and d set to their default range (–5 to 5)Set the sliders to $a = 1$, $b = 1$, $c = 0$, and $d = 0$. Then create another function $g(x) = a\,f(b(x - c)) + d$.

Part I:
Use your sketch to explore the following ideas and answer the questions:

1. With your sliders set to $a = 1$, $b = 1$, and $c = 0$, adjust the d slider and watch what happens to the graph.
    a. Next, repeat the process, observing how the graphs of each of the following functions change as you adjust the d slider:
        i. $f(x) = 2^x$
        ii. $f(x) = x^3$
        iii. $f(x) = \frac{1}{x}$
        iv. $f(x) = (x - 1)^2$
    b. Using complete sentences, precise language and one specific example, describe how adding or subtracting a constant value to or from ***any function*** affects its graph.

2. With the function f set back to $f(x) = x^2$ and your sliders set to $a = 1$, $b = 1$, and $d = 0$, adjust the c slider and watch what happens to the graph.
    a. Repeat the process, observing how the graphs of each of the following functions change as you adjust the c slider:
        i. $f(x) = \ln x$
        ii. $f(x) = \frac{x+3}{x-1}$
        iii. $f(x) = (x + 1)^2 + 2$
    b. Using complete sentences, precise language and one specific example, describe how adding or subtracting a constant value to or from ***the argument of any function*** affects its graph.

3. Do problems 5C #6 and 7 on p. 142 of your text.

Part II:

Use your Desmos sketch, explore the following ideas and answer the questions:

4. With the function f set back to $f(x) = x^2$ and your sliders set to $b = 1$, $c = 0$, and $d = 0$, adjust the a slider and watch what happens to the graph.
   a. Next, repeat the process, observing how the graphs of each of the following functions change as you adjust the a slider:
      i. $f(x) = \sin(x)$
      ii. $f(x) = e^x$
      iii. $f(x) = \frac{1}{x}$
      iv. $f(x) = (x-1)^2$
   b. Using complete sentences, precise language and one specific example, describe how multiplying ***<u>any</u> function*** by a constant value affects its graph. In particular, discuss the cases when $0 < |a| < 1$.

5. With the function f set back to $f(x) = x^2$ and your sliders set to $a = 1$, $c = 0$, and $d = 0$, adjust the b slider and watch what happens to the graph.
   a. Repeat the process, observing how the graphs of each of the following functions change as you adjust the b slider:
      i. $f(x) = \cos(x)$
      ii. $f(x) = \frac{x+3}{x-1}$
      iii. $f(x) = \ln x$
   b. Using complete sentences, precise language and one specific example, describe how multiplying ***the argument of <u>any</u> function*** by a constant value affects its graph. In particular, discuss the cases when $0 < |b| < 1$.

6. Consider the function $f(x) = x^2$ and the function $g(x) = 3f(2(x+1)) - 4$. Using complete sentences and precise language, describe the relationship between the graphs of f and g.

7. Consider the function $f(x) = \frac{3\sin\left(4x^2 - \ln x\right)}{x^5 - 4\pi^3\sqrt{x}}$ and the function $g(x) = \frac{1}{2}f\left(\frac{1}{3}x - 6\right) + 7$
   Using complete sentences and precise language, describe the relationship between the graphs of f and g.

8. Do problem 5D #7 on p. 144 of your text.

Part III:

Use your Desmos sketch, explore the following ideas and answer the questions:

9. In the first two parts of this Exploration, we explored the cases where $a \geq 0$ and $b \geq 0$. In this part, you should explore the cases for $a < 0$ and $b < 0$. Choose examples similar to those used in Parts I and II and explore the effect of a and b when they become negative. Using complete sentences and precise language, explain your findings. In particular, use mathematical language to describe the relationship between the graphs of $f(x)$, $-f(x)$ and $f(-x)$.

10. Do problems 5E #5-8 on p. 145 of your text.

Some hints about precise language:

Examples of words to use:

- Function, curve, argument of the function, value, the transformed function, shift, translate, stretch, compress, dilate, reflect, rotate, substitute, horizontally, vertically, extreme value, positive, negative, opposite, x or y intercept, x or y axis, root, zero, asymptote, increase, decrease.

Examples of words or phrases to avoid:

- Line (unless you're really talking about a line), slider, plug in, some, move, thing, stuff, about, roughly, a little bit, a lot, more, less.

IB Standard Level Mathematics: Exploring Transformations	**/7**
Name:	
Date Collected:	
A: Communication	**/4**
0: The work ***does not*** reach the standard described by the descriptor below.	
1: The work has ***some*** coherence. • Some coherence but not well organized, or some organization but not coherent. • Key explanations missing or incorrect • No examples • No evident understanding of ***why*** the transformation works the way it does	
2: The work has ***some*** coherence and shows ***some*** organisation. • Some mathematical and/or non mathematical explanations are missing • Coherent but not well organized, or well-organized but not coherent. • Some incomplete, incorrect or inappropriate explanations/examples • Little evident understanding of ***why*** the transformation works the way it does.	
3: The exploration is ***coherent*** and ***well organised***. • Most mathematical and/or non mathematical explanations are clear. • Aspects need clarification. • Lacks conciseness (could be huge detracting tables that should be in an appendix.) • Clear, relevant examples, mostly correct and general. • Explanations/examples do not cover all the cases. • Some evident understanding of ***why*** the transformation works the way it does.	
4: The exploration is ***coherent***, ***well organised***, ***concise*** and ***complete***. • Mathematical and/or non mathematical explanations are clear and concise. • Work is complete • Explanations are logically developed. • Examples & explanations cover all the cases and are correct and general. • Clear understanding of ***why*** the transformation works the way it does.	

B: Mathematical Presentation (includes mathematical vocabulary)	**/3**
0: The work ***does not*** reach the standard described by the descriptor below.	
1: There is ***some*** appropriate mathematical presentation. • Poor or minimal use of notation, terminology, and/or mathematical symbols. • Missed opportunities to show mathematical language.	
2: The mathematical presentation is ***mostly*** appropriate. • Inconsistency of terminology and/or variables. • Some key terms and variables defined	
3: The mathematical presentation is appropriate ***throughout***. • Key terms and variables explicitly defined. • Correct use of mathematical language, terminology, symbols and notation (no *, or ^) use of approximate ≈ instead of equal, appropriate use of subscripts etc. • Appropriate and varied forms of mathematical representation used	