Syllabus coverage

- Students must answer all the questions based on the option they have studied.
- Knowledge of the entire content of the option studied is required for this paper, as well as the core material.

Question type

- Questions require extended responses involving sustained reasoning.
- Individual questions may develop a single theme or be divided into unconnected parts. Where this occurs, the unconnected parts will be clearly labelled as such.
- Questions may be presented in the form of words, symbols, diagrams or tables, or combinations of these.
- Normally, each question reflects an incline of difficulty, from relatively easy tasks at the start of a question to relatively difficult tasks at the end of a question. The emphasis is on problem solving.

Mark allocation

- This paper is worth **60** marks, representing **20%** of the final mark. Approximately **15** marks are allocated to core material (or work of a similar level).
- Questions in this section may be unequal in terms of length and level of difficulty. Therefore, individual questions may not necessarily be worth the same number of marks. The exact number of marks allocated to each question is indicated at the start of each question. Each section is worth **60** marks, and the overall level of difficulty of each section should be the same.

Guidelines

Notation

Of the various notations in use, the IBO has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IBO notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

In a small number of cases, students may need to use alternative forms of notation in their written answers. This is because not all forms of IBO notation can be directly transferred into handwritten form. For vectors in particular the IBO notation uses a bold, italic typeface that cannot adequately be transferred into handwritten form. In this case, teachers should advise candidates to use alternative forms of notation in their written work (for example, $\vec{x}$, $\bar{x}$ or $\underline{x}$).

Students must always use correct mathematical notation, not calculator notation.

$\mathbb{N}$	the set of positive integers and zero, $\{0, 1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\mathbb{C}$	the set of complex numbers, $\{a + \mathrm{i}b \mid a, b \in \mathbb{R}\}$
i	$\sqrt{-1}$
z	a complex number
z^*	the complex conjugate of z
$\lvert z \rvert$	the modulus of z
$\arg z$	the argument of z
$\mathrm{Re}\, z$	the real part of z
$\mathrm{Im}\, z$	the imaginary part of z
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x \mid \quad \}$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$	the empty (null) set
U	the universal set
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of

$\subseteq$	is a subset of
A'	the complement of the set A
$A \times B$	the Cartesian product of sets A and B (that is, $A \times B = \{(a, b) \mid a \in A, b \in B\}$)
$a \mid b$	a divides b
$a^{1/n}$, $\sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$a^{1/2}$, $\sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geq 0$ then $\sqrt{a} \geq 0$)
$\lvert x \rvert$	the modulus or absolute value of x, that is $\begin{cases} x \text{ for } x \geq 0, x \in \mathbb{R} \\ -x \text{ for } x < 0, x \in \mathbb{R} \end{cases}$
$\equiv$	identity
$\approx$	is approximately equal to
$>$	is greater than
$\geq$	is greater than or equal to
$<$	is less than
$\leq$	is less than or equal to
$\ngtr$	is not greater than
$\nless$	is not less than
$[a, b]$	the closed interval $a \leq x \leq b$
$]a, b[$	the open interval $a < x < b$
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + ... + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + ...$

$\sum_{i=1}^{n} u_i$	$u_1 + u_2 + \ldots + u_n$
$\prod_{i=1}^{n} u_i$	$u_1 \times u_2 \times \ldots \times u_n$
$\begin{pmatrix} n \\ r \end{pmatrix}$	$\frac{n!}{r!(n-r)!}$
$f : A \to B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function under which x is mapped to y
$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$f \circ g$	the composite function of f and g
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	the second derivative of y with respect to x
$f''(x)$	the second derivative of $f(x)$ with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the n^{th} derivative of y with respect to x
$f^{(n)}(x)$	the n^{th} derivative of $f(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
e^x	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$	the natural logarithm of x, $\log_{\mathrm{e}} x$

sin, cos, tan	the circular functions
arcsin, arccos, arctan	the inverse circular functions
csc, sec, cot	the reciprocal circular functions
$\mathrm{A}(x, y)$	the point A in the plane with Cartesian coordinates x and y
$[\mathrm{AB}]$	the line segment with end points A and B
AB	the length of $[\mathrm{AB}]$
(AB)	the line containing points A and B
$\hat{A}$	the angle at A
$\mathrm{C\hat{A}B}$	the angle between $[\mathrm{CA}]$ and $[\mathrm{AB}]$
$\Delta\mathrm{ABC}$	the triangle whose vertices are A, B and C
$\boldsymbol{v}$	the vector $\boldsymbol{v}$
$\vec{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\boldsymbol{a}$	the position vector $\vec{\mathrm{OA}}$
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	unit vectors in the directions of the Cartesian coordinate axes
$\lvert\boldsymbol{a}\rvert$	the magnitude of $\boldsymbol{a}$
$\lvert\vec{\mathrm{AB}}\rvert$	the magnitude of $\vec{\mathrm{AB}}$
$\boldsymbol{v} \cdot \boldsymbol{w}$	the scalar product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\boldsymbol{v} \times \boldsymbol{w}$	the vector product of $\boldsymbol{v}$ and $\boldsymbol{w}$
$\boldsymbol{A}^{-1}$	the inverse of the non-singular matrix $\boldsymbol{A}$
$\boldsymbol{A}^{\mathrm{T}}$	the transpose of the matrix $\boldsymbol{A}$
$\det \boldsymbol{A}$	the determinant of the square matrix $\boldsymbol{A}$
$\boldsymbol{I}$	the identity matrix
$\mathrm{P}(A)$	probability of event A

$\mathrm{P}(A')$	probability of the event "not A"
$\mathrm{P}(A \mid B)$	probability of the event A given B
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
P_x	probability distribution function $\mathrm{P}(X = x)$ of the discrete random variable X
$f(x)$	probability density function of the continuous random variable X
$F(x)$	cumulative distribution function of the continuous random variable X
$\mathrm{E}(X)$	the expected value of the random variable X
$\mathrm{Var}(X)$	the variance of the random variable X
μ	population mean
σ^2	population variance, $\sigma^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
σ	population standard deviation
$\bar{x}$	sample mean
s_n^2	sample variance, $s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n}$, where $n = \sum_{i=1}^{k} f_i$
s_n	standard deviation of the sample
s_{n-1}^2	unbiased estimate of the population variance, $s_{n-1}^2 = \dfrac{n}{n-1} s_n^2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n-1}$, where $n = \sum_{i=1}^{k} f_i$
$\mathrm{B}(n,\ p)$	binomial distribution with parameters n and p
$\mathrm{Po}(m)$	Poisson distribution with mean m
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2

$X \sim \mathrm{B}(n, p)$	the random variable *X* has a binomial distribution with parameters *n* and *p*
$X \sim \mathrm{Po}(m)$	the random variable *X* has a Poisson distribution with mean *m*
$X \sim \mathrm{N}(\mu, \sigma^2)$	the random variable *X* has a normal distribution with mean μ and variance σ^2
Φ	cumulative distribution function of the standardized normal variable with distribution $\mathrm{N}(0,1)$
ν	number of degrees of freedom
χ^2	chi-squared distribution
χ^2_{calc}	the chi-squared test statistic, where $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e}$
$A \setminus B$	the difference of the sets *A* and *B* (that is, $A \setminus B = A \cap B' = \{x \mid x \in A \text{ and } x \notin B\}$)
$A \Delta B$	the symmetric difference of the sets *A* and *B* (that is, $A \Delta B = (A \setminus B) \cup (B \setminus A)$)
κ_n	a complete graph with *n* vertices
$\kappa_{n,m}$	a complete bipartite graph with one set of *n* vertices and another set of *m* vertices
$\mathbb{Z}_p$	the set of equivalence classes $\{0, 1, 2, \ldots, p-1\}$ of integers modulo *p*
$\gcd(a,b)$	the greatest common divisor of integers *a* and *b*
$\mathrm{lcm}(a,b)$	the least common multiple of integers *a* and *b*
$\boldsymbol{A}_G$	the adjacency matrix of graph *G*
$\boldsymbol{C}_G$	the cost adjacency matrix of graph *G*

Glossary of command terms

The following command terms are used without explanation on examination papers. Teachers should familiarize themselves and their students with the terms and their meanings. This list is not exhaustive. Other command terms may be used, but it should be assumed that they have their usual meaning (for example, "explain" and "estimate"). The terms included here are those that sometimes have a meaning in mathematics that is different from the usual meaning.

Further clarification and examples can be found in the teacher support material.

Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.
Calculate	Obtain the answer(s) showing all relevant working. "Find" and "determine" can also be used.
Find	Obtain the answer(s) showing all relevant working. "Calculate" and "determine" can also be used.
Determine	Obtain the answer(s) showing all relevant working. "Find" and "calculate" can also be used.
Differentiate	Obtain the derivative of a function.
Integrate	Obtain the integral of a function.
Solve	Obtain the solution(s) or root(s) of an equation.
Draw	Represent by means of a labelled, **accurate** diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Sketch	Represent by means of a diagram or graph, labelled if required. A sketch should give a general idea of the required shape of the diagram or graph. A sketch of a graph should include relevant features such as intercepts, maxima, minima, points of inflexion and asymptotes.
Plot	Mark the position of points on a diagram.
Compare	Describe the similarities and differences between two or more items.
Deduce	Show a result using known information.
Justify	Give a valid reason for an answer or conclusion.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions should not generally be "analysed" using a calculator.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.

Weighting of objectives

Some objectives can be linked more easily to the different types of assessment. In particular, some will be assessed more appropriately in the internal assessment (as indicated in the following section) and only minimally in the examination papers.

Objective	**Percentage weighting**
Know and use mathematical concepts and principles.	15%
Read, interpret and solve a given problem using appropriate mathematical terms.	15%
Organize and present information and data in tabular, graphical and/or diagrammatic forms.	12%
Know and use appropriate notation and terminology (internal assessment).	5%
Formulate a mathematical argument and communicate it clearly.	10%
Select and use appropriate mathematical strategies and techniques.	15%
Demonstrate an understanding of both the significance and the reasonableness of results (internal assessment).	5%
Recognize patterns and structures in a variety of situations, and make generalizations (internal assessment).	3%
Recognize and demonstrate an understanding of the practical applications of mathematics (internal assessment).	3%
Use appropriate technological devices as mathematical tools (internal assessment).	15%
Demonstrate an understanding of and the appropriate use of mathematical modelling (internal assessment).	2%

Internal assessment details 20%

The purpose of the portfolio

The purpose of the portfolio is to provide students with opportunities to be rewarded for mathematics carried out under ordinary conditions, that is, without the time limitations and pressure associated with written examinations. Consequently, the emphasis should be on good mathematical writing and thoughtful reflection.

The portfolio is also intended to provide students with opportunities to increase their understanding of mathematical concepts and processes. It is hoped that, by doing portfolio work, students benefit from these mathematical activities and find them both stimulating and rewarding.