page 1: **1**. 1, 6.684, 10.747, 12.102, 12.467,… approaches $\sqrt[3]{1996} = 12.5908\ldots$ **2**. 17576000 possible plates **3**. (2,90) (2,–270) $(\sqrt{2},135)(\sqrt{2},-225)(10,-36.9)(10,323.1)(\sqrt{50},81.9)$ $(\sqrt{50},-278.1)(\sqrt{50},261.9)(\sqrt{50},-98.1)$ **4**. 5 sec; $y = 336$, speed 64 **5**. 34.8 more minutes **6**. (–3.00,7.42) **7**. some of the y-intercepts are 0.535, 1.231, 2.832, 6.515, …; $-570 \le \theta \le 970$; parametric equations are $x = 2^{t/360}\cos t$, $y = 2^{t/360}\sin t$ **8**. sin(x+3) not equal to sin(x) + sin(3); property holds for $f(x) = mx$ and also for $f(x)$=sin(120x) **9**. a sinusoidal model is not appropriate

page 2: **1**. 39 minutes **2**. some x-intercepts –2.280, –1.316, 1, 1.732, 3, …; some y-intercepts –2.615, –1.510, 1.147, 1.987, …. **3.** –1, 2, 2.5; $y = (x+2)(7x-22)(x-8)$ **4a**. –0.5, 1.5, 3 **4b**. –1, 3, 6 **4c**. –3, 1, 4 **4d**. –1/m, 3/m, 6/m **5**. symmetry in y-axis; $f(x) = \cos x$, $f(x) = x^2$ **6**. $r = 2 + r\sin\theta$ and $\theta = 90$ **7**. the point is $(\sqrt[3]{1996}, \sqrt[3]{1996})$, and $\sqrt[3]{1996}$ is the limit of the recursive sequence defined by f **8**. yes, as long as x_0 is positive **9a**. 5 sec; 240 ft **9b**. 48 fps **9c**. (x, y) = (96, 336); 80 fps

page 3: **1**. period 6; sin 60x; cos 60x **2**. $16y = 7x^2$ fits the three extreme points, but it also fits (3, 3.94) **3a**. r = 10 and h = 12.73 make total surface 1428.3 sq cm **3b**. total area = $2\pi r^2 + \frac{8000}{r}$ **3c**. r = 8.60 and $h = 17.21$ make the total area 1394.9 sq cm **4**. the graph of f + g is asymptotic to the graph of f at both ends, and also asymptotic to the y-axis **5**. x, y, y, x **7.** $n > 50$ **8**. $n > \log(p)/\log(5/6)$

page 4: **1**. half-turn symmetry at the origin; $f(x) = \sin x$, $f(x) = x^3$ **2**. 3 + 2i and 3 – 2i **4**. real; pure imaginary (if it is not zero); the line x = 2; the line $x = y$ **5a**. 5 **5b**. 1 **5c**. $\sqrt{13}$ **5d**. $\sqrt{13}$ **6**. symmetric in the line $x = 90$; $f(x) = \sin x$, $f(x) = |x - 90|$

page 5: **1.** $r = 12/(2 - \sin\theta)$ is an ellipse **2**. $4x^2 + 3(y-4)^2 = 192$; a=8, $b = \sqrt{48}$, c=4, c/a = 1/2 **4**. 245 more minutes **5**. 3 + 4i is on the circle of radius 5 centered at the origin **6**. 5 + 2i; –4 + 3i; 8 + 6i; usual component-by-component addition **7**. symmetry in line x = 2 **8**. 70/256 = 27.3%; 182/256 = 71.1% **9**. unit vectors, or the usual parametrization of the unit circle; cis 53.1; cis 120; cis 73.7 **10**. $2\,\text{cis}\,72 = 0.618 + 1.902i$; $3 + 4i = 5\,\text{cis}\,53.1$

page 6: **1**. $f(x) = f(x+72)$, $f(x) = f(144 - x)$ **2**. $(x-2)(x+2)$; $(x-2i)(x+2i)$ **3**. 50% **4**. yes; the point (–a, –b) is on the graph whenever (a, b) is **5a**. $\sqrt{x-1}$ **5b**. $4x^2 + 9$ **5c**. $\sqrt{34}$ **5d**. 14 **5e**. $\sqrt{x^2+9}$ **6**. power functions $p(x) = x^m$ and $q(x) = x^n$ work, as do inverse-function pairs **7**. the FD tickets are worth 25 cents and the PD tickets are worth 20 cents **8**. **a**, **c**, **d** approach 1; **b** and **e** approach no limit; answer depends on where x_0 is in relation to 0 and 1.8 **9**,**10**. $p(q(x))$ is periodic if q is, and is perhaps periodic if p is **11**. all angles are about 80.1 deg **12**. –1

page 7: **1**. 1/36; most likely sum is 7 (prob = 6/36) **2**. 5/72 **3**. $f(x) = 2^x$ **4a**. $f(x) = f(80-x)$ or $f(40-u) = f(40+u)$ **4b**. $f(x) = -f(80-x)$ or $f(40-u) = -f(40+u)$ **5**. counter-clockwise quarter-turn **6**. $n > 14$ **7**. $n > \log((2-p)/p)/\log(2)$, assuming $0 < p < 2$ **8**. annual yields: 1.09, 1.0938, 1.0942 **9**. 2.00, 2.613, 2.715, 2.718 (essentially the best rate possible) **10**. 1.09417… **11a**. 0 **11b**. 100/3 **11c**. –1

page 8: **1**. 13; $a^2 + b^2$ is never negative **2**. $x^7 + 7x^5 + 21x^3 + 35x + 35x^{-1} + 21x^{-3} + 7x^{-5} + x^{-7}$ **3**. 1/3838380 **4**. average payoff per ticket is $2.605 **5a**. all multiples of 60 **5b**. 4 **5c**. –2.5 **5d**. –2, 0, 2 **6a**. –1, 1.25, 2 **6b**. –4, 5, 8 **6c**. –1, 8, 11 **6d**. –10, 12.5, 20 **6e**. –1/2, 4, 11/2 **7**. 11/36 **8**. 59.5 deg **9**. the sum is 2/3 when $x = -1/2$; no x makes the sum 1/5 **10**. $x > 10000$ **11**. $x > \tan(p)$ **12**. $f(x) = \log x$ **13**. the slope of the curve $y = 2^x$ at its y-intercept is $\lim_{h \to 0} \frac{2^h - 1}{h} = 0.693...$

page 9: **1**. 1/84; add at least 11 red apples **2**. the polar grid consists of circles and rays **4**. 5 cis 53.13; $\sqrt{2}$ cis 45; $5\sqrt{2}$ cis 98.13 **5**. $\sqrt{5}$ = 2.23607 **6**. might converge to $-\sqrt{5}$ = –2.23607 **7**. $p(4) = 1/4$; the probability function $p(n) = {}_nC_3\left(\frac{1}{2}\right)^n = \frac{n(n-1)(n-2)}{6 \cdot 2^n}$ is meaningful for all nonnegative integers n, though it can be graphed for any n; $\lim_{n \to \infty} p(n) = 0$ **8**. four; two real roots **9**. 56; 12cis 56 **10**. $k = 2.594$; $k = 2.705$; $k = \left(1 + \frac{1}{n}\right)^n$; thus $b = e = 2.71828\ldots$; Q approaches (0,1) as n grows

page 10: **1**. 1; reciprocal slopes **2**. 16 + 0i **3**. the positive x-intercepts are $\ldots, b^{-1}, 1, b, b^2, \ldots$, they form a geometric sequence; the negative x-intercepts are $\ldots, -b^{3/2}, -b^{1/2}, -b^{-1/2}, -b^{-3/2}, \ldots$, which also form a geometric sequence; if $b < 1$, the curve spirals toward the origin as θ increases **4**. 3.5; 7.0; 10.5; 3.5n **5**. thirty **6**. $f(180-x) = f(x)$; $f(360+x) = f(x)$; $f(180-x) + f(x-180) = 0$ **7**. e^2; e^r **8**. $q(4) = 5/16$; for any positive integer n, $q(n)$ is $1 - \left(\frac{1}{2}\right)^n - n\left(\frac{1}{2}\right)^n - {}_nC_2\left(\frac{1}{2}\right)^n = 1 - \frac{n^2+n+2}{2}\left(\frac{1}{2}\right)^n$; $\lim_{n \to \infty} q(n) = 1$ **9**. any number greater than 1/2 can be a sum

page 11: **1**. the circle whose Cartesian equation is $(x-2)^2 + y^2 = 4$; (4, 0); negative r-values! **2**. $(r;\ \theta) = (r;\ \theta + 360) = (r;\ \theta + 720) = (-r;\ \theta + 180)$ **3** $2 + i = \sqrt{5}\text{cis}\,26.6$, $3 + 4i = 5\text{cis}\,53.1$, $2 + 11i = \sqrt{125}\text{cis}\,79.7$, $-7 + 24i = 25\text{cis}\,106.3$ **4**. $(2\text{cis}\,30)^{18} = -262144$ **5a**. high (–1,15); low (1,–21); x = –2, 1/2, 3/2 **5b**. high (3,13); low (7,1) **6**. 60 fps; 32 fps; 68 fps **7**. 720 ft; $720 - 64k - 16k^2$ ft; $64k + 16k^2$ ft; 64+16k fps **8**. –64 fps is the vertical component of the instantaneous velocity at time 2 **9**. the velocity vector [60,–224] makes a 75.0-degree angle with the ground **10**. both are correct; the limit is $\sqrt{e}$

page 12: **1a**. 1/6 **1b**. 1/3 **2**. i^{83} should have been $-i$ **3**. lines $x = 0$ and $y = x/2$ are asymptotes **5**. the same spiral **6**. i; -1; $-i$; 1 **7**. $r = \frac{1}{3}(24 + r\cos\theta)$ **8**. $ac - bd + (ad + bc)i$ yields $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ and $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$ **9**. 20.4 min from the time it was poured **10**. the asymptotes $x = -1$ and $y = 2$ are evident **11a**. $2i$, $-2i$, 1, -1 **11b**. -1 has multiplicity two

page 13: **1**. 0.739085… **2**. reflection in real axis; real; real **3**. $0.5 - 0.5i$ **4**. $\text{cis}(-\theta)$ **5**. 25/51 = 49.0% **6**. the possible red counts are 1, 3, 6, 10, 15, … **7a**. $x^2(3-x)$ **7b**. x^2 is never negative **7c**. (1, 2) **7d**. (2,4) is local maximum **8**. evidence for #7 **9a**. $-1+i\sqrt{3}$ and $-1-i\sqrt{3}$ **9b**. 2, $-1+i\sqrt{3}$, and $-1-i\sqrt{3}$ **9c**. $1 - i$ **10**. 36/49

page 14: **1**. slope of the second graph (2.20) is twice the slope of the first (1.10) **2**. 55 **3**. 55 **4**. $x = 2/3$; $x = 1995/1996$; $-1 < x < 1$ **5**. $-1 < x < 1$; all numbers greater than 1/2; asymptote $x = 1$ **6**. slope is 1 **7**. limit = 1 = slope **8**. $\frac{1}{2}\left(x + \frac{5}{x^2}\right)$ works slowly; $\frac{1}{3}\left(2x + \frac{5}{x^2}\right)$ works rapidly **9**. $9\sin(5x)$ or $4\sin(\pi x/36)$ **10**. slope of first graph is m times slope of second graph at any two corresponding points (same x-value) **11**. $(x+yi)(x-yi)$

page 15: **1**. asymptotes are $x = 1$ and $y = x + 1$; range of y-values: all numbers except those between 0 and 4 **2**. all sums greater than 1/2; $\frac{1}{1-2x} = \sum_{n=0}^{\infty}(2x)^n$ for $-\frac{1}{2} < x < \frac{1}{2}$ **3**. $b = 16$ **4**. the limit is $\frac{1}{1-m}$ for $-1 < m < 1$ **5**. 1344904/3838380 = 35.0% **6**. **c** only **7**. the asymptotes of $3(y+4)^2 - x^2 = 12$ have slope $\pm 1/\sqrt{3}$, so $\theta = 30$ and $\theta = 150$ give trouble; the eccentricity is 2; vertices at (0, –2) and (0, –6); foci at (0, 0) and (0, –8) **8a**. 2640 **8b**. 1 **8c**. $\dfrac{1-\left(\frac{i}{2}\right)^{5281}}{1-\frac{i}{2}}$, which is approximately $0.8+0.4i$ **9**. $(x+y)^{5280}$ **10a**. 80.1° **10b**. $3^{1/4}$ times $3^{(\theta-90)/360}$ is $3^{\theta/360}$ **10c**. 80.1°

page 16: **1.** $y = x^4$; $x^2(4-x^2)$; $(1-x^2)(4-x^2)$; no **2**. (1.001,1.003003001); slope of graph at (1,1) is 3 **3**. $(x-1)(x^2+x+1)$ and $(x-2)(x^2+2x+4)$; $\lim_{x\to 2}\frac{x^3-8}{x-2} = \lim_{x\to 2} x^2 + 2x + 4 = 12$ **4**. 2; m **5**. slope = ln 2 **6**. 120 **7**. $x_n + y_n i \approx (x_{n-1} + y_{n-1} i)\cdot \text{cis}\,30$ **8**. $f(x) = \cos(22.5x - 22.5)$ is one example; other lines of symmetry are $x = 1 + 8n$ **9**. $\frac{1}{2}(-i \pm 3)$ **10**. $7.6 + 8.2i$ **11**. 2; m; 3; 1 **12**. the graph of $y = x^2 + x + 1$ is punctured at (1,3); $x \neq 1$; $y \neq 3$ and $3/4 < y$

page 17: **1**. 1669536/3838380 = 43.5% **2**. the probability of two correct is 695640/3838380 = 18.1%; the probability of three correct is 119680/3838380 = 3.1%; the probability of four correct is 8415/3838380 = 0.2% **3**. $r = \theta/600$ **4**. $\frac{1}{1-r} = 1 + r + r^2 + r^3 + \cdots$ **5**. as long as the seed value is neither 0 nor 1, a 3-cycle is obtained **6**. $n = 16$ **7**. vert asymptotes at the zeros of cos; range of secant: numbers that are not between –1 and 1 **8**. vert asymptotes $x = 1$ and $x = -1.4$; hori asymptotes $y = 0$ and $y = \frac{1}{2}$; y-int at $-\frac{1}{2}$ **9**. $n(n-1)(n-2)\cdots(n+1-r)$ **10**. –414720 **11**. the graph of $y = x - 1$ is punctured at $(-1,-2)$; $x \neq -1$; $y \neq -2$

page 18: **1**. all are conic sections that have a focal point at the origin; parabola, ellipse, hyperbola (the eccentricity is k) **2**. 4845; 2025; 2025/4845 = 41.8% **3**. 6.96% **4**. 28/323 = 8.67% **5**. your sequence is probably eventually periodic: …, 4, 2, 1, 4, … **6**. horizontal asymptotes $y = 0$ and $y = 1$; half-turn symmetry at (0, 1/2) **7**. 110 yards beyond the starting line **9**. $p = 110$ **10**. vert asymptote at $x = -3$; y-int $-2/3$ **11**. $1 + i$

page 19: **1**. –1/2 **2**. two 63.43-deg angles created **3**. x must be between –1 and 1, inclusive; mode does not matter **4**. upper half of the unit circle **5**. t must be between 0 and 180, inclusive (in degree mode) **6**. 2.079; 5.545; 5.545 **7**. 3.466 = 5(0.693) **8**. e^3; e^3; e^a (aha!) **9**. cis 45 = $\frac{1}{2}(1+i)\sqrt{2}$; cis 225 = $-\frac{1}{2}(1+i)\sqrt{2}$ **10**. [1,2], [2,3], [63245986,102334155]

page 20: **1**. most such sequences approach one of the square roots of $3 + 4i$, which are $2 + i$ and $-(2 + i)$; one sequence that does not approach either root is seeded by $2 - 4i$ **2**. 48 **3**. ten **4**. 15992 **5**. 1/54979155 **6**. 650.8 mi; each ticket worth $5.45 **7**. 0 **8a**. 1 is most likely distance (62.5%) **8b**. 1.875 is expected distance **9**. d-sequence same as f-sequence; r-sequence approaches 1.618… 10. no linear asymptotes; even function; $|x| \neq 1$; $y \geq 4$ and $y \neq 5$

page 21: **1**. the one-sided limits are –1 and 1 **2**. $e \approx 2.7183$ **3**. $y = 1$ is a horizontal asymptote **4**. other roots are –4 and $2 - 2i\sqrt{3}$ **5**. horizontal asymptote $y = 1$ **6a**. $\pi/2$ **6b**. $-\pi/2$ **6c**. m **6d**. 2 **7**. $y = x - 1 - \frac{5x}{x^2+1}$ has $y = x - 1$ as an oblique asymptote, and half-turn symmetry about (0, –1) **8**. 1 is slope of the tangent line at (2, –2) **9**. the value of the limit depends on x: $2x - 3$ is the slope of the tangent line at $(x, f(x))$ **10**. the slopes are 1, 0.5, 0.25, 0.2, 0.125; product of x and slope is 1 **11**. asymptotes: $x = -1$ and $y = x - 3$; dom: $x \neq -1$; range: all y *except* $-4 - \sqrt{12} < y < -4 + \sqrt{12}$

page 22: **1**. mag = 1.012574…; angle = 57.2838 deg, or 0.99979 radian **2**. mag = 1.0050; angle = 57.29387 deg, or 0.99997 radian **3**. mag = 1; angle = 1 radian **4**. cis 2; cis(–1); $\text{cis}\,\pi = -1$; $\text{cis}\,\theta$ **5**. 1; 2.5; the first is the sum of probabilities of obtaining x heads in five flips of a fair coin, for x = 0, 1, …, 5; the second sum is the expected number of heads **6**. 0 **7**. 49.66% **8**. $a = -7$ and $b = 24$ **9**. always period 5

page 23: **1**. the one-sided limits do not agree when x is an integer **2**. 233, a member of the Fibonacci sequence **3**. $4 + i$ and $-(4 + i)$ **4**. horizontal asymptotes $y = 1$ and $y = -5/3$; half-turn symmetry at $(\log_b 3, -1/3)$ **5**. 5.81%; on the sixth or seventh roll **6a**. 0 **6b**. 0 **7**. $2x - 5$ is the same as $2(x - 2.5)$, so one can compress by a factor of 2 first and then slide right 2.5 **8**. $\frac{1}{6}\sqrt{3} = 0.289$ is the slope of the curve $y = \sqrt{x}$ at the point where $x = 3$ **9a**. e **9b**. e **9c**. $e^{1/3}$ **9d**. e

page 24: **3**. $u = x + h$ **4**. slope = $1.099y$ **5**. limit = ln 3 = 1.0986… **6**. $3 = (1 + xh)^{1/h}$; as h approaches 0, x approaches ln 3 **7**. $h = 1/n$ **8**. $(\ln 0.96)\left(0.96^x\right)$; $-300(\ln 0.96)\left(0.96^t\right)$; $-300(\ln 0.96)\left(0.96^t\right)$ **9**. looks like $y = \cos x$ **10**. looks like $y = -\sin x$

page 25: **1**. $-32t$ **2**. $[-\sin t, \cos t]$ **3**. it takes 0.01256 sec to go from 0.6 to 0.61, which is 0.796 ups **4**. $\angle BAO \approx \frac{\pi}{2}$ when h is very small **5**. $-300(\ln 0.96)\left(0.96^t\right)$ **6**. limit is $-\sin t$ **8**. yes, velocity is $-\sin t$ in any quadrant

page 26: **1a**. $E'(x) = b^x \ln b$ **1b**. $p'(x) = 3x^2$ **2**. looks like $y = \cos x$ **3**. $g'(x) = nx^{n-1}$ **4**. $L'(x) = m$

page 27: **1**. $f(x) = (b/a)x$; in general, the graph joins (0,0) to (a,b), so that all slopes are greater than 1; $f'(x)$ is the local stretch factor **2abc**. 0.08624; 0.00885; 0.00089 **3**. 14 kg **4**. $20(0.85)^2 = 14.45$ kg **5a**. 20.00, 19.40, 18.82, 18.25, 17.71, 17.17, 16.66, 16.16, 15.67, 15.20, 14.75 **5b**. –0.60, –0.58, –0.57, –0.56, –0.54, –0.51, –0.50, –0.49, –0.47, –0.45; multiply these by 10; they are differences **5c**. all entries are –0.3; kg per min per kg **6**. $x = a + h$

page 28: **1**. 12.96 mhos/sec **2**. velocity is $x'(t) = 4 - 2t$; speed is $|x'(t)| = |4 - 2t|$ **3**. difference quotients **4**. $a - b$; slopes are $y'(x) = \frac{1}{2\sqrt{x}}$ **5**. yes, $\frac{1}{2}x^{-1/2}$ conforms **6a**. when h is very small, $\angle BCA \approx \angle CAP = \angle OPD = \frac{\pi}{2} - t$ and $\angle CBA \approx \frac{\pi}{2}$ **6c**. $PQ \approx h(\sec t)^2$ **7**. the slopes are all positive

page 29: **1**. $R'(x) = -x^{-2}$ **3**. $20\left(1 - \frac{0.3}{n}\right)^n$ **4**. $\lim_{n\to\infty} 20\left(1 - \frac{0.3}{n}\right)^n = 14.816...$ kg of salt **5**. $e^{-0.3} = \lim_{n\to\infty}\left(1 - \frac{0.3}{n}\right)^n = 0.741$ **6**. $S(t) = 20e^{-0.3t}$; at any instant, 30% of the available salt is removed per hr **7**. –0.259; –25.9 **8**. average speed **9**. the minimum speed should be 0

page 30: **1**. group terms so that $\lim_{h\to 0}\frac{\sin(h)}{h}$ and $\lim_{h\to 0}\frac{\cos(h)-1}{h}$ appear **2**. $S(t)=20(0.92)^t$ or $S(t)=20e^{-0.0834t}$ **3**. 6 and 1/6; reciprocals **4**. 1/12 **5**. $1/\exp'(a)=1/\exp(a)=1/b$ **6a**. [0,1] **6b**. [–1,0] **6c**. [–0.8,0.6] **6d**. $[-\sin\theta,\cos\theta]$ **7**. [0,10]; [–10,0]; [–8,6]; $[-10\sin\theta,10\cos\theta]$; $(x,y)=(\cos 10t,\sin 10t)$; linear motion defined by velocity vector [–8,6] **8**. 3(0.582)=1.752 **9**. $3m$ **10**. $3\cos(3a)$ **11**. $f(t)=k\cdot e^t$

page 31: **1**. $f'(a)$ **2**. $-2x^{-3}$; yes **3**. replace x by x–5 to obtain $-2(x-5)^{-3}$; translation does not alter tangent slopes **4**. $g'(x)=f'(x-h)$ **5**. both limits are 1 **6**. $f(t)=k\cdot e^{-0.42t}$ **7**. 5.18 **8.** factor k out of the difference quotient **9**. km **10**. $kf'(0)$ **11**. $40\pi/9=13.96$; $(40\pi/9)\left(\sqrt{3}/2\right)=12.09$

page 32: **1**. $2=\sec^2\frac{\pi}{4}$, so $P=\left(\frac{\pi}{4},1\right)$ and $Q=\left(1,\frac{\pi}{4}\right)$ **2**. $F'(x)=2ax+b$; when $x=-\dfrac{b}{2a}$, $F(x)$ is extreme **3**. $S(x)=300/x$ for $0<x<50$; if $a<b$ then $S(b)<S(a)$ **4**. -2.0161; -2.0764; –2.0826; $-\frac{300}{144}=-2.08333$ **5**. 4, 4, 4; –8.0645, –8.3056, –8.3306 approach $-25/3$ **6**. $\frac{dy}{dt}=\frac{dx}{dt}\cdot\frac{dy}{dx}$ **7**. $y'(3)=x'(3)S'(12)=-25/3$ **8**. –41.6667 **9**. $A'(r)=2\pi r$; $V'(r)=4\pi r^2$; yes **10**. $7x^6$; $\cos\frac{\pi}{6}=\frac{1}{2}\sqrt{3}$; $2^a\ln 2$

page 33: **1**. 76 = 72+4 **2**. $k'(t)=3t^2+10t$ **4**. nondifferentiable at $x=0$ because $\lim_{h\to 0}\frac{|h|-0}{h-0}$ does not exist (two one-sided limits do not agree) **5**. $3(x^2+5)^2 2x$, or $6x^5+60x^3+150x$ **7a**. T is continuous for all nonnegative x **7b**. one-sided derivatives at x=41200 are 0.15 and 0.28 **7c**. five values **8a**. $b=e^{-1}$ and $\ln b=-1$ **8b**. slope at $\left(x,e^{-x}\right)$ is the opposite of the slope at $\left(-x,e^{-x}\right)$ on $y=e^{-x}$ **8c**. $y'=-e^{-x}$ **9**. $1/m$ **10**. $\frac{1}{n}a^{(1/n)-1}$

page 34: **1a**. $g(2.1)=-2.5$; $g(1.85)=-5.0$ **1b**. $g(2.1)=-3.92$; $g(1.85)=-2.87$ **2**. $k'(x)=g'(f(x))f'(x)$ **3**. (4.1)(21.2)(0.009) rad = 44.8 deg **4a**. S is a decreasing function **4b**. –0.86580; $\frac{dS}{dw}\Delta w=-\frac{100}{27}\cdot\frac{24}{100}=-0.88889$ **5**. 9; 24; $0.2416\approx 24(0.01)$ **6**. $S=\frac{300}{16t^2}$, so $\frac{dS}{dt}=-\frac{75}{2}t^{-3}$, which is –88.89 fps when $t=0.75$ **7**. 0 deg; 90 deg

page 35: **1**. derivatives provide infinitely many different examples; all have the indeterminate form $\frac{0}{0}$ **2**. yes **3**. the second formula is correct **5a**. (6.00,6.00) **5b**. (5.20,7.40) **5c**. (6.72,3.36) and (–1.36,–0.68) **5d**. (–1.04,–0.75) and (4.63,7.69) **7a**. $f'(x)=3x^2+3^x\ln 3$ **7b**. $M'(\theta)=24\sec^2(3\theta)$ **7c**. $H'(u)=3(\sin u)^2\cos u$ **8**. $P'(x)=\frac{m}{n}x^{(m/n)-1}$

page 36: **1a**. $F(x)=x^4+c$ **1b**. $g(t)=2\sin(5t)+c$ **1c**. $S(u)=\frac{1}{2}e^u-\frac{1}{2}e^{-u}+c$ **2**. all **3a**. 1 sec **3b**. 96 cm/sec; $y'(t)=48\pi\sin 2\pi t$ is 48π when t is 0.25 **4a**. if $t = m/n$ is rational, and if x is the probability that the molecule persists for time interval $1/n$, then $q=x^n$, so $q^{1/n}=x$ and the desired probability is $x^m=\left(q^{1/n}\right)^m=q^t$ **4b**. apply previous probability **4c**. $A'(t)=A(t)\ln q$ **4e**. 0.3000 is instantaneous (relative) rate of salt loss **5**. no; Kyle will get 1/2 instead of 0

page 37: **1a**. $0.999879^2=0.999758$ **1b**. $0.999879^{1/2}=0.999939$ **1c**. 0.999879^y **1d**. $A(t)=0.999879^t A(0)$ **1e**. $\frac{A'(t)}{A(t)}=\ln(0.999879)=-0.000121007$ is the instantaneous rate at which carbon-14 atoms decay; the probability that a specific atom will decay during a (short) time interval Δt is $0.000121007\,\Delta t$ **2**. apply the Chain Rule to $P(x)=\left(x^k\right)^{-1}$ **3a**. $f'(x)=2x-2x^{-3}$ **3b**. $g'(t)=3-5\cos t$ **3c**. $L'(x)=\frac{-x}{\sqrt{4-x^2}}$ **3d**. $P'(t)=-4\pi\sin\pi t$ **4**. nondifferentiable at $x=0$ because $\lim_{h\to 0}\frac{h^{1/3}-0}{h-0}$ does not exist; $g(x)=x^{1/2}$ or $g(x)=x^{1/5}$ **5**. $\frac{1}{2}=\cos\frac{\pi}{3}$, so $P=\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$ and $Q=\left(\frac{\sqrt{3}}{2},\frac{\pi}{3}\right)$ **6**. $Q'(x)=18x^{-5/2}$ **7**. $W(x)=\frac{2}{3}x^{3/2}+c$ **8**. one could substitute $x=t^3$

page 38: **1**. right side looks like $\frac{d}{dx}$ was applied to sin(3x), but that is not what the left side says; removing the first '3' from the right side would make the equation correct **2**. $f(t)=k\cdot e^{0.5t}$ **3**. $D_x\left(x^u\right)=ux^{u-1}$ and $D_u\left(x^u\right)=x^u\ln x$ **4**. $m=n/x$ approaches ∞ as n does, because x is positive **5**. the answer is the reciprocal of $\lim_{m\to\infty}\left(\frac{m+1}{m}\right)^{m+1}$ **8**. 1 **9**. 1 **10**. limiting radius is 1; limiting angle is 1

page 39: **1a**. after 9 seconds of travel, the object is moving in the negative direction at 1.8 meters/sec **1b**. approximately 6.6 meters to the right of P **2**. acceleration **3**. no **4a**. $W(x)=\frac{1}{43}x^{43}+c$ **4b**. $S(u)=\frac{1}{43}\left(7+u^2\right)^{43}+c$ **4c**. $F(u)=u^2+c$ **4d**. $g(t)=(\sin t)^2+c$ **5**. 0.936; $\sqrt{1-k^2}$ **7**. the slope of arcsin at (a,b) is $\frac{1}{\cos b}=\frac{1}{\cos(\arcsin a)}=\frac{1}{\sqrt{1-a^2}}$ **8**. treating h as a constant, $\frac{dV}{dr}=2\pi rh$; treating r as a constant, $\frac{dV}{dh}=\pi r^2$; in the latter case, V is proportional to h **9**. $f(x)=x^2-8x+19$ and $f(x)=-x^2+4x+1$ are two more

page 40: **1**. 192.08 – 187 = 5.08; of this, 3.4 is due to increased width and 1.65 is due to increased height **2**. in 0.001 second, the area increases by 187.050503 – 187 = 0.050503; of this, 0.034 is due to increased width and 0.0165 is due to increased height, so 0.000003 is due to neither, and this is 0.006% of the total change **3**. $A' = WH' + HW'$ **5a**. 0.04 ft **5b**. 8 ft/sec; 0 ft/sec **5c**. no; no **5d**. $-4\pi \sin 400\pi t$ **5e**. $-4\pi \sin 400\pi t \sin \pi x$ equals $-2\pi\sqrt{2} \sin 400\pi t$ when $x = 0.25$ **6**. $f'(x) = n(g(x))^{n-1} g'(x)$ **7a**. $\frac{dy}{dx} = \sin x + x\cos x$ **7b**. $\frac{dy}{dx} = x + 2x \ln x$ **7c**. $\frac{dy}{dx} = \cos^2 x - \sin^2 x$ **7d**. $\frac{dy}{dx} = \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4-x^2}}$

page 41: **1**. c = 13/3 and a = 9/20; the common slope at x = 1 is –3 **2**. $\arctan' x = \frac{1}{1+x^2}$ **3**. $R(t) = 3960e^{kt}$ **4**. $\frac{dy}{dx} = -\frac{x}{y}$ **6a**. (0,0) and (–1.5,1.5) **6b**. (–0.8,1.5) and (1.7,–3.4) and (6.0,3.0) **6c**. (6,3) and (5.32,2.66) **6d**. (–1.1,1.6) and (2.2,–3.7) and (5.8,3.1) **7**. there are no gaps in the graph, but the tangent slopes at the endpoints are undefined

page 42: **1**. $f(x) = 4e^{(3-x)/2}$ or $f(x) = 2 + 2e^{3-x}$ **2**. $y = 1 + e^{-2}x$ and $y = -1 + \ln 2 + \frac{1}{2}x$ **3a**. $f'(t) = 2te^{-t} - t^2e^{-t} = \left(2t - t^2\right)e^{-t}$ **3b**. $g'(u) = \frac{5}{2}u^{3/2}$ **3c**. $M'(x) = 2^{\sin x}(\cos x)\ln 2$ **3d**. $R'(t) = -\frac{1}{2\sqrt{t}} \sin\sqrt{t}$ **4a**. $f'(t) = 3t^2 - 18t + 15 = 3(t-1)(t-5)$ **4b**. for t < 1 or for 5 < t **4c**. for 1 < t < 5 **4d**. f(1) = 16 and f(5) = –16 **4e**. –8 ups; 8 ups; 12 ups **5**. b–1 is the relative change in the population during time interval from t to t+1 **6**. during interval from t to t+h, the average rate of change of the population, as a fractional part of the population, is $\frac{b^h - 1}{h}$ **7**. A is the population size at time t = 0; $m = \ln b = f'(t)/f(t)$ is instantaneous rate of growth of the population, expressed as a fractional part of the population **8**. the implicit slope formula is $\frac{dy}{dx} = -\frac{18x}{2y}$, which equals –4 at (4,9) **9a**. $0 < x < 4$ **9b**. 0.624; –1.816; the maximum volume V occurs for some x-value between 1.5 and 2.0 **9c**. V(5/3) = 2450/27 = 90.741 is the maximum

page 43: **1p**. f is decreasing; f' is negative, f' is increasing **1q**. f is increasing; f' is positive, f' is increasing **1r**. f is increasing; f' is positive, f' is decreasing **1s**. f is decreasing; f' is negative, f' is decreasing **2**. for $0 \le t \le 30$, an exponential approaching 350 from below; for $30 \le t \le 60$, an exponential approaching 70 from above; spliced continuously at t=30 **3**. $Q' = \frac{gf' - fg'}{g^2}$

page 44: **1**. $\frac{dy}{dx}=\frac{1-x}{y+3}$; –3/4; 0; 4/3 **2a**. the time to drain the tank is known to be 125/20 = 6.25 min, so average descent rate is 5.76 in/min **2b**. not a prism **2c**. in a fifth of a minute, 4 cu ft of water leave, which means that $-4 \approx 30\Delta y$, or $-2/15 \approx \Delta y$ **2d**. $-20\Delta t \approx 30\Delta y$, so $(-2/3)\Delta t \approx \Delta y$ **2e**. $\frac{dy}{dt}=-\frac{2}{3}$ when A = 30; $\frac{dy}{dt}=-\frac{20}{A}$ in general **3a**. $\frac{dy}{dx}=\frac{2}{(x+1)^2}$ **3b**. $\frac{dy}{dx}=\frac{1}{(\cos x)^2}$ **3c**. $\frac{dy}{dx}=-\frac{84}{x^3}$ **3d**. $\frac{dy}{dx}=-\frac{1}{2x^{3/2}}$ **3e**. $\frac{dy}{dx}=\frac{1-x^2}{(1+x^2)^2}$ **4**. $f(x)=4-2\sin(x-3)$ is another **5**. $g(a)f'(a)+f(a)g'(a)$ is the Product Rule

page 45: **1a**. 20/7 = 2.857 fps **1b**. 35 feet **1c**. 35/8 = 4.375 fps **2b**. P is nondifferentiable and discontinuous at positive integers; $P'(x)=0$ when x is a positive noninteger **2c**. $P(x)=39+24n$ for $n<x\le n+1$ **3**. $E(t) = G(t)$, and both ratios equal ln b **4**. the graph is defined for $x < 1$; the line $y = -x$ is tangent to the graph at the origin **5**. the probability that the atom will persist during a t-second interval is q^t; the rate of change of this decreasing function, as a fractional part of the function itself, is ln q = ln(1–p), which is approximately –p when p is a small positive number **6**. Product Rule; Power Rule; Quotient Rule **7**. slope is $e\ln 2 = 1.884$; tangency point is $(\log_2 e, e)$

page 46: **1a**. pulse holding steady at 153 bpm **1b**. pulse is 162; increasing 10 bpm per minute **1c**. pulse holding steady at 180 bpm **2**. $f(t)=ke^{-0.12t}$ **3a**. $\frac{dx}{dt}=1+2\cos t$ **3b**. $0\le t<\frac{2\pi}{3}$ or $\frac{4\pi}{3}<t\le 2\pi$ **3c**. $\frac{2\pi}{3}<t<\frac{4\pi}{3}$ **3d**. $t=\frac{2\pi}{3}$, $x=\frac{2\pi}{3}+\sqrt{3}$; $t=\frac{4\pi}{3}$, $x=\frac{4\pi}{3}-\sqrt{3}$ **3e**. 1 **3f**. $\frac{2\pi}{3}+4\sqrt{3}$ units in 2π sec is $\frac{1}{3}+\frac{2\sqrt{3}}{\pi}$ ups; greatest speed is 3 ups **4a**. $a+b+c=2$ **4b**. $b=-2-2a$ and $c=4+a$ **4c**. $a=1/2, b=-3, c=9/2$; x = 3 **5**. $\frac{dy}{dx}=-\frac{1}{\sin y}$ **6**. $\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$ **7**. $S=\left(36\pi V^2\right)^{1/3}$; $\frac{dS}{dV}=\left(\frac{32\pi}{3V}\right)^{1/3}$

page 47: **1a.** water leaves the tank only through the (exposed) water surface **1b.** ft/min **1c.** because $\Delta V \approx A\Delta y$, it follows that $V' = A\cdot y'$, so $y'=-0.12$ **1d**. $y(t)=3-0.12t$, so $y(25)$ = 0 **2**. $f(x)=12\sqrt{3}\,x^{-3/2}$ **3**. $x=2y+9$ **4**. $y=36-0.12t$ intersects the t-axis at $t=300$; $y=36e^{-0.12t}$ never intersects the t-axis **5**. $y=k-0.12t$; $y=k\cdot e^{-0.12t}$ **6**. $x'=\frac{z}{x}z'=\frac{1250}{750}\cdot 66=110$, which is too fast

page 48: **1a**. no; $f(3)$ has to be greater than $f(-1)$ because $f'(x)$ is positive for every x between -1 and 3 **1b**. no; the tangent line to the graph of f at $(-1,3)$ has slope 2 and thus goes through (3,11); the slopes of the graph are smaller than 2 for every x between -1 and 3, however, so the graph lies below its tangent line **1c**. $x = 4$ **1d**. $x = -3$; min value is about 1/3, because $f'(x) \approx 2-\frac{1}{2}(x+1)^2$ seems to be true for $-3 \le x \le -1$, which implies that $f(x) = 2x-\frac{1}{6}(x+1)^3+5$ **2**. Chain Rule says that $D(\cos t + i\sin t) = i(\cos t + i\sin t)$; match real and imaginary parts **3**. (c) is the derivative of (a); (d) is the derivative of (b); using the points (0,0), (3,0), and (1,1) on graph (a) and (0,0) and (2,0) on graph (c), the formulas $y=\frac{1}{2}x^2(3-x)$ and $y' = 3x-\frac{3}{2}x^2$ can be found; the inverted V of graph (d) is made from $y = 2x$ and $y = 4-2x$, spliced together at (1,2); use the points (1,1) and (2,2) on graph (b) to splice together $y = x^2$ and $y = 4x - x^2 - 2$ at (1,1) **4**. y-intercept is $y=\frac{1}{2}+a^2$ **5**. $\frac{dy}{dx}=\frac{y^2}{1-2xy}$

page 49: **1**. the slopes $y' = 6x-3x^2$ are all positive for x between 0 and 2; the largest slope is 3, at $x = 1$ **2**. extreme points occur when the derivative changes sign; quadratic functions change sign twice or not at all; $y = x^3$ has no extreme points **3**. derivative is a quadratic function, so extreme point is midway between its zeros **4**. $1/a$ **5**. optimal height is $8\sqrt{3}$ **6a**. $f'(t) = -5e^{13-5t}$ **6b**. $g'(t) = 1$ **5c**. $M'(t) = v'(t)e^{v(t)}$ **7ab**. find where $f'(x)$ is maximal **8**. 5/8 cm/min **9a**. $f''(x) = 2$ **9b**. $f''(z)=\frac{1}{z}$ **9c**. $f''(u) = -4\cos 2u$ **9d**. $f''(t) = e^{-t^2}\left(4t^2-2\right)$ **10b**. any positive $z < e^{-1}$ will do **11**. any t greater than $\frac{1}{\sqrt{2}}$ will do

page 50: **1**. abbreviation of $\frac{d}{dx}\left(\frac{df}{dx}\right)$ **2**. $x=\frac{1}{6}a$ gives maximal volume $\frac{2}{27}a^3$ **3**. $2\arctan\frac{1}{\sqrt{2}} = 70.5°$ **4**. $\frac{1}{t\ln b}$ **5**. $T(x)=\frac{1}{30}\sqrt{100+x^2}+\frac{1}{50}(20-x)$; $T'(x)=\frac{x}{30\sqrt{100+x^2}}-\frac{1}{50}$; min time is $T(7.5)=\frac{2}{3}$ **6**. $\frac{dx}{dt}=4t^2(t-3)$ and $\frac{d^2x}{dt^2}=12t^2-24t$ **6a**. ($t = 0$, $x = 3$) and ($t = 3$, $x = -24$) **6b**. x increases when $3 < t$; x decreases when $t < 0$ and when $0 < t < 3$ **6c**. $t = 0$ and $t = 2$; locally extreme velocity **6d**. $-24 \le x \le 8$ **6e**. $-16 \le x' \le 64$ **7** $t = \cos^{-1} 0.8 = \sin^{-1} 0.6$; velocity = $[-5\sin t, 15\cos t]$ is $[-3,12]$ at (4,9); slope is -4 **8**. $f'(x) = -\frac{1}{x}\sin\ln x$; dom f is $0 < x$; range f is $-1 \le f(x) \le 1$; $g'(x) = -\tan x$; dom g is $|x-2n\pi| < \frac{\pi}{2}$; range g is $g(x) \le 0$

page 51: **1**. 2.005 osc/sec; $f''(t) = -158.76f(t)$ **2**. it's a local maximum **3**. $f'(x) = e^{-x}x^{n-1}(n-x)$ changes from positive to negative at $x = n$ **4**. it's a local minimum **5**. it's an inflection pt **6a**. $T(x)=\frac{1}{p}\sqrt{a^2+x^2}+\frac{1}{q}\sqrt{b^2+(c-x)^2}$; $T'(x)=\frac{x}{p\sqrt{a^2+x^2}}-\frac{c-x}{q\sqrt{b^2+(c-x)^2}}$ **6b**. $\frac{\sin PJO}{\sin PDN}=\frac{p}{q}$ **6c**. $T'(x)$ changes sign change from negative to positive; diminishing x makes the first fraction decrease and the second fraction increase **7**. 32.2 deg east of south; 49.3 km; 2.15 hrs