May 2016 subject reports

MATHEMATICS SL TZ1

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2016 examination session the IB has produced time zone variants of Mathematics SL papers.

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 53	54 - 65	66 - 77	78 - 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 17	18 - 20

The range and suitability of the work submitted

There were a wide variety of explorations with a wide range of quality this session and most clearly related to areas that interested the students. As in the past, certain themes appeared repeatedly — sports, games (board games, video games, gambling), the golden ratio, codebreaking, movies, music. Some were proper explorations and others were questionable in the sense that one could tell from the original topic choice that little mathematics would be involved. Many explorations involved regression although the understanding differed greatly. A lot of these simply used technology to generate the regression equations and graphs but did not go further to demonstrate understanding or to suggest why a particular regression equation was chosen. Others attempted to use chi-squared testing, and were generally not successful, as neither the student nor the teachers seemed to understand the process fully. A number of explorations involved throwing objects or kicking balls and modeling and analyzing the paths of travel. These were usually simplistic models but they often satisfied the requirements and could still score well. The worst cases were explorations that became simple surveys of information and/or a summary of facts about a topic. Students often had a hard time showing much personal engagement or demonstration of understanding. These included logic problems, game theory and topics such as the Golden ratio, the Fibonacci numbers, the origin of e and card counting in blackjack.

A number of explorations from individual schools were very similar to each other. This could be the same topic, the same format of work but with different numbers or following the same process throughout the exploration as though the students had been provided with a template. It was evident in many cases that the teacher may have overly guided the students rather than allowing them to explore their interests.

In addition some students tackled challenging mathematics more suited to higher level. Whether they have actually understood what they have written about was not always clear.

Candidate performance against each criterion

Criterion A

Most work was organized to some degree, with a relevant introduction, a rationale, an aim and some kind of conclusion, but coherence issues were occasionally a problem. Some students would make assumptions about the reader and leave many steps unexplained. Aims were often vague, for example ‘I aim to find out more about…’ and this made the exploration confusing at times since the conclusion could not be linked back to the aim. Most candidates recognize the need for citation and reference lists but there is still a problem with images and data not being cited where they occur in the paper (this is not an issue that is penalized in this criterion but it does speak to organization and is a requirement of all explorations). Weaker students have difficulty writing papers that qualify as complete; stronger ones have more trouble making their explorations concise. There is a balance between those two descriptors which is clearly challenging for students at this level. For instance, it is unnecessary to repeat lengthy calculations. It is inappropriate, in a mathematical exploration, to provide “how to” guides on using specific pieces of software.

Criterion B

This criterion was generally well understood by teachers and students. Most students were able to select appropriate mathematical representations and used terminology in an appropriate manner. A good standard of computer technology was demonstrated here in producing graphs and equations. Some common issues were poor or missing labels on graphs, terms left undefined, and use of calculator notation in the text. A few students put a good deal of effort into phrasing their work in such a way as to avoid having to include mathematical notation. This is not conducive to scoring well in mathematical presentation.

Criterion C

Many students made a point of mentioning a personal interest in their topic. Sometimes this was all the personal engagement present. The fact that a student is interested in the general topic of the paper does not mean that he or she will necessarily score highly in this criterion. Evidence of further engagement must be present in the paper itself. A common “investigation/textbook problem” is unlikely to achieve the higher levels on criterion C unless the students extends this further by asking themselves ‘What if...’ Simply choosing a topic that is above the level of the course does not, by itself, qualify as outstanding personal engagement, although learning new mathematics is one aspect that can contribute to this criterion. In some modelling and statistical explorations obvious opportunities for displaying greater independence, creativity and or personal/interest by designing and collecting their own primary data, rather than using standard secondary data sets, were overlooked.

Criterion D

Some students made an excellent effort to reflect regularly on results as they appeared and what each new set of calculations or derivation means in the context of the aims. The best papers use that reflection to guide the next steps taken in the analysis. Many left reflections to the end and even included a sub-heading for this. Reflection should be more than just restating the conclusions found. These conclusions were often limited or superficial. Rather than just summarizing results students can also consider limitations and possible extensions of their investigations, relative strengths and weaknesses of approaches taken, and alternative perspectives on the topic. Critical reflections included discussions of specific mathematical results in the context of the topic.

Criterion E

Some students presented topics that were clearly not commensurate with mathematics SL and were taken from the prior learning topics, or chose topics that would lead to difficult mathematics beyond their level of understanding. Regression models were often treated with technology and showed little understanding as it was not clear why a particular regression model was chosen or was appropriate. Another common shortcoming is that complicated formulas were used and applied but without any evidence to support student understanding of how and why these formulas actually worked. The top level of 6 is still difficult to reach, and teachers have sometimes awarding this level to work in which the student has significant errors which have apparently gone undetected. This is a concern. It is surely difficult to thoroughly check the explorations for mathematical errors when every student has a different topic, but it is nevertheless important that teachers make a good faith effort to do so. One of the key differentiators between attainment levels is the degree of understanding that is demonstrated within the student work. It is not the degree of difficulty but the level of understanding that is assessed in this criterion.

Recommendations for the teaching of future candidates

In general students need more exposure to exploratory mathematics before they are assigned the IA Exploration. As concepts are introduced in class there may be room for short explorations or activities that allow students to learn what it means to explore and also learn the intent of each criterion. This may also encourage students to seek out new and novel topics instead of falling back on well-known topics from a mathematical text or other sources. Before settling on topics, students should be afforded the opportunity to read some of the better-scoring papers in the Teacher Support Material. Teachers can challenge students to explain what mathematics they will do on their own before approving a topic. Teachers need to ensure that students meet internal deadlines and have obtained relevant feedback to their first drafts. Explorations that are poorly planned often do not score well against the assessment criteria. Students could teach the new mathematics they are attempting to explore/learn to their peers to assess how successful their understanding is as well as what type of questions could arise. They could then use this in their explorations.

Some additional recommendations regarding the criteria:

- The notion of coherence needs better explaining to teachers and students. Work should not appear "out of nowhere". Students should not leave any results or methods in the work without comment and interpretation.
- The value of having a clear aim should be highlighted as this can make for a better exploration overall.
- It is recommended that schools explicitly instruct their students in the use of one of the many tools, free or otherwise, that produce correct mathematical notation and graphs.
- Candidates need to be encouraged to handle mathematics within their capacity of understanding. Use of higher mathematical concepts with minimum understanding does not help them score well.

Further comments

Teachers should be strongly encouraged to complete the background information. It is also essential for them to indicate their markings on the student work. Similarly, it would be helpful if they give elaborate comments against each criterion. Teacher comments on the work also need to be clear and preferably legible.

Schools in which teachers provide specific comments both on the forms 5/EXCS and on student work seem far less likely to have their marks changed by the moderators. Teachers who do this appear to understand the criteria better and the annotations make it easier for moderators to understand the teachers' reasoning.

However, a number of schools continue to submit samples with no or very few annotations on the work. With reference to past feedback forms, such repeat offenders certainly made the moderation much more difficult.

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 35	36 - 46	47 - 56	57 - 67	68 - 90

General comments

The areas of the programme and examination which appeared difficult for the candidates

- Application of the double angle rule in context
- Recognising a quadratic form and the application of the discriminant in context
- Application of the laws of logarithms in context
- Communication in a 'show that' question
- Recognising and applying calculus properties to specified and unspecified functions

The areas of the programme and examination in which candidates appeared well prepared

- Functions (composite, inverse, sketching and transformations)
- Using Venn diagrams and simple probability
- Application of arithmetic sequences and series
- Statistics
- Working with basic quadratics
- Basic quadratics

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: functions

This question was successfully answered by most candidates. The inverse notation was sometimes mistakenly interpreted as derivative or reciprocal.

Question 2: probability with Venn diagram

This question was well done by most candidates. In part (b), the intersection $P(A \cap B)$ was sometimes overlooked, incorrectly using $P(A \cup B) = P(A) + P(B)$ instead.

Question 3: trigonometric graph

Almost all candidates correctly stated the amplitude but then had difficulty finding the correct period. Few students faced problems in sketching the graph of the given function, even if they had found the wrong period, thus indicating a lack of understanding of the term 'period' in part a)ii). Most sketches were good although care should be taken to observe the given domain and to draw a neat curve.

Question 4: arithmetic sequences and series

Most candidates recognized that the series was arithmetic but many worked backwards using $n = 200$ rather than creating and solving an equation of their own to produce the given answer. Almost all students answered (b) correctly.

Question 5: quadratics

As a 'show that' question, part a) required a candidate to independently find the answers. Again, too many candidates used the given answers (of 3 and -6) to show that the two zeros were 3 and -6 (a circular argument). Those who were able to recognize that the x-coordinate of the vertex is -1.5 tended to then use the given answers and work backwards thus scoring no further marks in part a). Answers to part b) were more successful with a good variety of methods used and correct solutions seen.

Question 6: area of triangle and double angle rule

Many candidates earned only the first method mark for substituting the sides into the area formula. Others realized that there was a double angle but made the error $\sin 2\theta = 2\sin\theta$ instead of using $\sin 2\theta = 2\sin\theta\cos\theta$. Those candidates who found a value for $\cos\theta$ typically were able to answer the question correctly.

Question 7: using discriminant

There was a minor issue with the domain of the function, but this did not affect any candidate. The question was amended for publication.

Most candidates recognized the need to set the functions equal to each other and many rearranged the equation to equal zero. Few students then recognized the quadratic form and the need to find the discriminant. Those who did use the discriminant generally completed it correctly.

Question 8: statistics

Generally, candidates were very successful with this question, appearing to move easily between the three different representations of data. The main conceptual errors appeared in part d) where a percentage of 100 was found (instead of 80) and in part e) where the new standard deviation was often given as 20.5. Arithmetic errors seemed to be the other factor, with a surprising number of candidates finding in part c) that $800-745=15$.

Question 9: integration and logarithms

Part a) was well answered. In part b) most candidates realised that integration was required but fewer recognised the need to use integration by substitution. Quite a number of candidates who integrated correctly omitted finding the constant of integration. In part d) many candidates showed good understanding of transformations and could apply them correctly, however, correct use of the laws of logarithms was challenging for many. In particular, a common error was $\frac{\ln 27}{\ln 3}=\ln 9$.

Question 10: differentiation and tangents

Part a) was relatively well answered – the obvious errors seen were not using the chain rule correctly and simple fraction calculations being wrong.

In parts b) and c) it seemed that the students did not have a good conceptual understanding of what was actually happening in this question. There was lack of understanding of tangents, gradients and their relationship to the original function, *g*. A working sketch may have been beneficial but few were seen and many did a lot more work than required.

In part d) although candidates recognized $h(x)$ as a product and may have correctly found $h(1)$, they did not necessarily use the product rule to find $h'(x)$, instead incorrectly using $h'(x)=f'(x)\times g'(x)$. It was rare for a candidate to get as far as finding the equation of a straight line but those who did usually gained full marks.

Recommendations and guidance for the teaching of future candidates

Teachers need to be familiar with the whole Mathematics SL guide, not just the syllabus content. Teachers in particular should be familiar with the prior learning topics, command terms, notation list, formula booklet and approaches to teaching and learning. Candidates are expected to apply their knowledge in non-routine situations and this needs to be practiced throughout their course of study.

Communication is an important part of undertaking mathematics and candidates have definitely improved in this matter over recent years. However, 'show that' questions, which appear regularly in past papers, are still not answered well. Candidates need to *obtain* the required result, not to *use* the required result, as this would be working backwards.

All topics need to be taught in such a way that not only mathematical processes are practiced but also that concepts can be generalized, understood and *applied*. It was clear that many students knew a rule but were completely lost when asked to apply it in a context, e.g. when the double angle rule appears in an expression for area.

Some basic exam practices are worth remembering: label all parts and subparts well, cross out any earlier attempts not used to find the answer when replaced with other working and draw graphs neatly and clearly (not drawn lightly which are difficult to read) being mindful of key features and the domain. Be aware that in the long response questions proceeding parts can often be used to help find later answers. Candidates should not be wasting time trying to do difficult computations on the non-calculator paper - if the calculations are difficult, an earlier error has probably been made.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 39	40 - 49	50 - 59	60 - 69	70 - 90

The areas of the programme and examination which appeared difficult for the candidates

Candidates in this session had difficulties in the following areas of the programme:

- Graphical solutions to equations
- Bearings
- Manipulating logarithms and exponents
- Interpreting the solution to an inequality in abstract and real life situations
- Differentiation and integration of trig functions
- Interpreting velocity – time graphs
- Radian measure

The areas of the programme and examination in which candidates appeared well prepared

For students who were well prepared, there was ample opportunity to demonstrate a high level of knowledge and understanding on this paper. The following areas of the programme were handled well by most students.

- Geometric sequences and series
- Binomial expansion
- The normal distribution
- Areas between curves
- Triangle trigonometry including sine and cosine rules
- Use of scalar product to find the angle between vectors
- Discrete probability distributions and expected value

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: normal distribution

There was a mixed response to this question: candidates either very confidently answered each part, or were completely lost. Most were able to shade the correct region in part (a) and in part (b), answers were not always left to two decimal places. It was not uncommon to see candidates converting to standard normal form in part (c) before using their calculators, suggesting that they were not only unfamiliar with the meaning of the command term "write down" but were unclear as to what the given information was actually asking for.

Question 2: area between curves

Candidates often did not make the connection between parts (a) and (b). The extraordinary number of failed analytical approaches in part (a) and correct use of the GDC to find the limits in part (b) suggests that candidates are equating the command term "solve" to mean use an algebraic approach to solve equations or inequalities, instead of their GDC. Many candidates appeared to interpret part (a) as something they should do by hand and often did not recognize that their answer to part (a) were the limits in part (b). Quite a few candidates failed to interpret a GDC solution of $x = 5 \times 10^{-14}$ correctly as $x = 0$ and others found the solution $x = 1.74$ as the only solution, ignoring the second intersection point until part (b).

Question 3: triangle trigonometry

Some candidates tackled this question very competently, whilst others struggled to obtain a correct answer even for part (a) which would generally be regarded as prior learning. Parts (b) and (c) were generally answered well, even with follow through from an incorrect angle in part (a). Weaker candidates assumed the triangle to be a right triangle and attempted to use Pythagoras to find AC. One of the most significant errors seen throughout this question was with candidates substituting an angle in degrees into a calculator set in radian mode.

Question 4: binomial theorem

Many candidates approached this question using an appropriate and efficient method to identify the required term. While many of those who were successful in (a) were also successful in (b), a significant number of candidates did not realize that multiplying their answer in (a) by $5x$ would give them the term in x^7. This led to an attempt to find a binomial expansion, which was generally unsuccessful. Some candidates continue to be unable to distinguish between the "coefficient" and the "term" and lost a point as a result.

Question 5: linear regression

This turned out to be one of the more challenging questions on the paper. Although many candidates correctly described the linear correlation in part (a), a surprisingly large number of candidates were unable to do so. Part (b) was not well done with many candidates unable to transfer their knowledge of exponentials and/or log manipulation to the question. After rewriting the line of best fit as $= e^{-0.12t+4.67}$, candidates were neither able to apply the rules for exponentials correctly nor were they familiar with the method of comparing coefficients to find the answer. There were numerous failed attempts of trying to estimate points from the graph and substitute these into the equation, while others arbitrarily chose values for t in an effort to set up a system of equations, the latter having some measure of success.

Question 6: geometric sequences and series

The majority of candidates did well on this question although identifying the common ratio was not always as easily done and some candidates lost marks as a result of using an inappropriate method such as $^{8}/_{4}$. Other candidates correctly guessed the value of r or did so by showing that if $r = 2$, then the ratio of the fourth term to the first term would be 8. It was also disappointing to see some candidates use an incorrect formula for the sum of the first n terms of a geometric series: a common error seen was $2557.5 = \frac{u_1(2^{10}-1)}{10-1}$, which led to the wrong answer of 22.5.

Question 7: application of exponential functions

Part (a) was generally done well, with many candidates able to find the value of k correctly and to interpret its meaning. Lack of accuracy was occasionally a concern, with some candidates writing their value of k to 2 significant figures or evaluating $\ln(0.9)$ incorrectly.

Few candidates were successful in part (b) with many unable to set up an inequality or equation which would allow them to find the condition on t. Some were able to find the value of t in decades but most were unable to correctly interpret their inequality in terms of the least number of whole years. While a solution through analytic methods was readily available, very few students attempted to use their GDC to solve their initial equation or inequality.

Question 8: discrete probability distributions

Candidates generally found parts (a), (b)(i) and (c) of this question the most straightforward and those who recognised the binomial distribution in (b)(ii) were usually able to obtain the required solution using their GDCs. Part (d)(i) proved to be more problematic with many candidates identifying one possible way of having two breakdowns (usually 1A and 1B), but not recognising three ways of having two breakdowns. Furthermore, many were not able to successfully calculate the probability of two breakdowns on one machine (and none on the other). The conditional probability in (d)(ii) was generally recognised though and those who showed their working in full were able to score follow through marks in this part.

Question 9: kinematics

This question was not well done throughout. Analytical approaches were almost always unsuccessful as a result of poor integration and differentiation skills and many of the errors were a result of having the GDC in degree mode. In (a), most candidates recognized the need to integrate v to find the displacement, although a significant number differentiated v. Of those that integrated, many assumed incorrectly that the initial displacement was the value of the constant of integration. Some candidates integrated $|v|$ and obtained no marks for an invalid approach. In the case where a correct definite integral was given, it was disappointing to see many candidates try to evaluate it analytically rather than using their GDC. In part (b), many candidates did not read the question carefully and gave the two occasions, in the given domain, where the particle was at rest. In part (c), many candidates did not appreciate that velocity is a vector and that the particle would change direction when its velocity changes sign. Consequently, many candidates gave the incorrect answer of four changes in directions, rather than the correct two direction changes. Part (d), was done very poorly, with candidates struggling to differentiate sine and cosine correctly and to evaluate their derivative. As with question 3, many candidates worked with the incorrect angle setting on their calculator. Few candidates attempted part (e). Of those that did, many attempted to find the largest local maximum of the graph rather than least local minimum as they did not recognise speed as $|v|$.

Question 10: vector geometry

The majority of candidates had little difficulty with parts (a) and (c). The most common error in both these parts were unforced arithmetic errors and occasional misreads of the vectors. In part (b), candidates who were successful used a variety of different approaches, and it was pleasing to see the vast majority of these being well reasoned, however, there were numerous unsuccessful responses including those who attempted to use the given vector to work backwards. A lack of appropriate vector notation often meant that ideas were not always clearly communicated. The majority of candidates struggled to make any progress in (d), with very few realizing that simple right-angled trigonometry could be used. Few were able to successfully express CD in terms of OC which was required to show the given result. Very few candidates attempted (d)(ii), with many unable to make the connection with results found in previous parts of the question.

Recommendations and guidance for the teaching of future candidates

Be sure to read the subject reports each session which continue to repeat recommendations regarding skills that are absolutely essential for Mathematics SL but are still not well understood or applied.

It is essential that both teachers and students are familiar with the Mathematics SL guide, especially the syllabus content (including prior knowledge), command terms and notation list, so that students are adequately prepared for this examination.

This paper has revealed that many candidates have not been exposed enough to a variety of questions on the major syllabus topics. There is a need for a teaching of concepts that involves more different approaches to solving problems in different possible contexts.

There were a number of questions in this paper where candidates were poorly prepared in the proper use of their GDC. Candidates should be aware of when an analytical approach is necessary and when one using their GDC will suffice. In general, for Paper 2, once an equation has been set up, there is little reason why its solution should not come directly from the GDC.

Teachers are encouraged to spend time with their students going through a solution using technology, so that students are aware of what is required to demonstrate full understanding and to gain all the available marks. In preparation for this paper, students should be given the opportunity to practice, in particular, changing the angle mode from one question to another within a test, and given feedback where incorrect GDC use is noted.

Students should be given more guidance on how to work multi-part questions and encouraged to look for links with earlier parts in a question.

Candidates do not have a clear understanding of how to round answers correctly to three significant figures and are losing marks as a result of inaccurate answers. They should also be advised to work with a minimum of four significant figures in the case of non-exact values, and only round to three significant figures at the end of a question part.

Candidates should be entered at an appropriate level for IB Mathematics exams. It appeared that the level of this course was too difficult for some of the candidates.