Quadratics – Practice Problems

1. (a) Factorize $x^2 - 3x - 10$.

(b) Solve the equation $x^2 - 3x - 10 = 0$.

Working:

Answers:
(a) ....
(b) ....

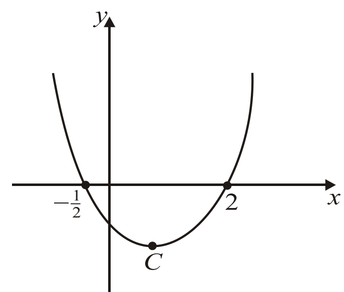
(Total 4 marks)

2. The diagram represents the graph of the function

$$f : x \longmapsto (x - p)(x - q).$$

(a) Write down the values of p and q.

(b) The function has a minimum value at the point C. Find the x-coordinate of C.



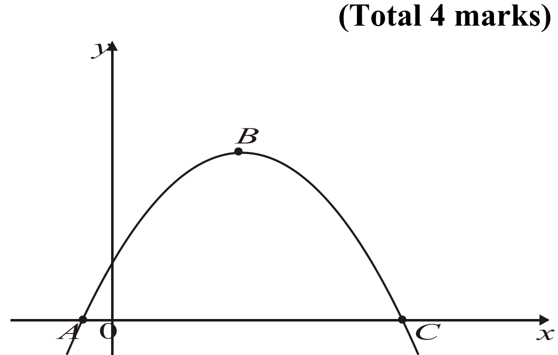
Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

3. The diagram shows the parabola $y = (7 - x)(1 + x)$. The points A and C are the x-intercepts and the point B is the maximum point.

Find the coordinates of A, B and C.



Working:

Answer:
....

(Total 4 marks)

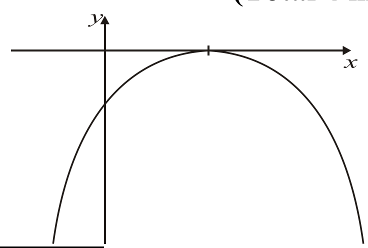
4. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x.
Find the value of k.

Working:

Answer:
....

(Total 4 marks)

5. The diagram shows the graph of the function $y = ax^2 + bx + c$.



Complete the table below to show whether each expression is positive, negative or zero.

Expression	positive	negative	zero
a			
c			
$b^2 - 4ac$			
b			

(Total 4 marks)

6. (a) Express $f(x) = x^2 - 6x + 14$ in the form $f(x) = (x - h)^2 + k$, where h and k are to be determined.
(b) Hence, or otherwise, write down the coordinates of the vertex of the parabola with equation $y = x^2 - 6x + 14$.

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

7. A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2,\ t \geq 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 17 metres.

(2)

(c) At a later time the ball is **again** at a height of 17 metres.
(i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.
(ii) Solve the equation **algebraically**.

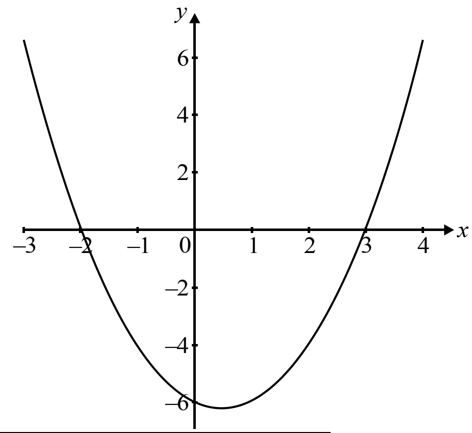
(4)

(d) (i) Find $\frac{dh}{dt}$.
(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).
(iii) Find **when** the ball reaches its maximum height.
(iv) Find the maximum height of the ball.

(7)

(Total 15 marks)

8. The diagram shows part of the graph with equation $y = x^2 + px + q$.
The graph cuts the x-axis at -2 and 3.



Find the value of

(a) p;

(b) q.

Working:

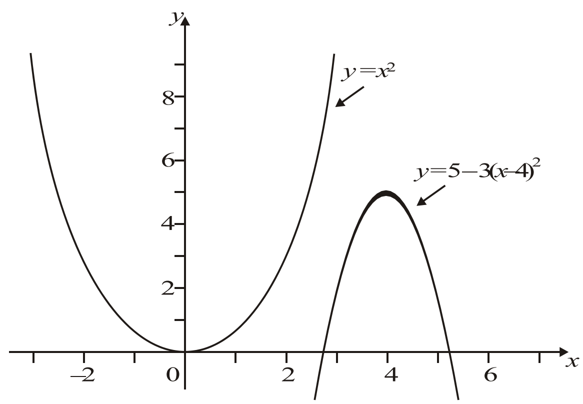
Answers:

(a) ....

(b) ....

(Total 4 marks)

9. The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x - 4)^2$.



The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x - 4)^2$ by these transformations.

A reflection in the line $y = 0$ **followed by**
a vertical stretch with scale factor k **followed by**
a horizontal translation of p units **followed by**
a vertical translation of q units.

Write down the value of

(a) k;

(b) p;

(c) q.

Working:

Answers:

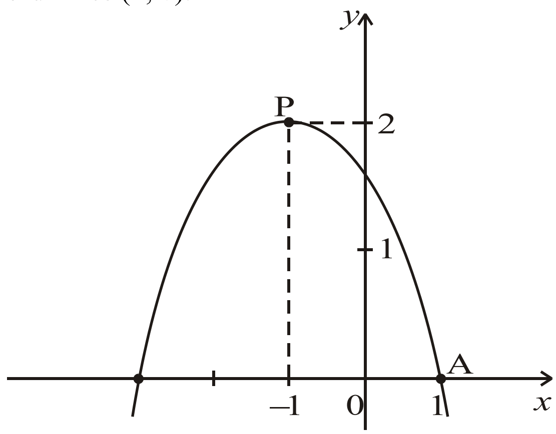
(a) ....

(b) ....

(c) ....

(Total 4 marks)

10. The diagram shows part of the graph of $y = a(x - h)^2 + k$. The graph has its vertex at P, and passes through the point A with coordinates (1, 0).



(a) Write down the value of
  (i) h;
  (ii) k.

(b) Calculate the value of a.

Working:

Answers:
(a) (i) ....
  (ii) ....
(b) ....

(Total 6 marks)

11. Consider the function $f(x) = 2x^2 - 8x + 5$.

(a) Express $f(x)$ in the form $a(x - p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b) Find the minimum value of $f(x)$.

Working:

Answers:
(a) ....
(b) ....

(Total 6 marks)

12. A family of functions is given by

$$f(x) = x^2 + 3x + k, \text{ where } k \in \{1, 2, 3, 4, 5, 6, 7\}.$$

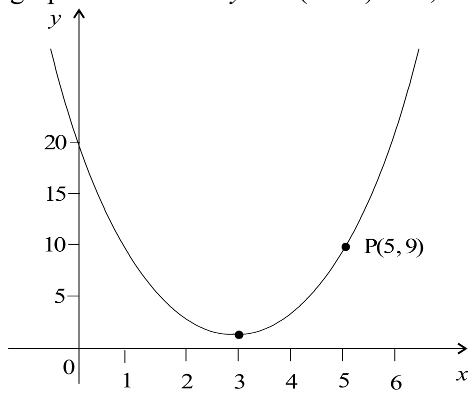
One of these functions is chosen at random. Calculate the probability that the curve of this function crosses the x-axis.

Working:

Answer:
....

(Total 6 marks)

13. The diagram shows part of the graph of the curve $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$.



(a) The vertex is at the point (3, 1). Write down the value of h and of k.

(2)

(b) The point P (5, 9) is on the graph. Show that $a = 2$.

(3)

(c) Hence show that the equation of the curve can be written as

$$y = 2x^2 - 12x + 19.$$

(1)

(d) (i) Find $\frac{dy}{dx}$.

A tangent is drawn to the curve at P (5, 9).

(ii) Calculate the gradient of this tangent.

(iii) Find the equation of this tangent.

(4)

(Total 10 marks)

14. The equation $kx^2 + 3x + 1 = 0$ has exactly one solution. Find the value of k.

Working:

Answer:

....

(Total 6 marks)

15. The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.

(a) Write $f(x)$ in the form $(x - a)^2 + b$.

(b) Find the inverse function f^{-1}.

(c) State the domain of f^{-1}.

Working:

Answers:

(a) ....

(b) ....

(c) ....

(Total 6 marks)

Quadratics – Practice Problems - Markscheme

1. (a) $x^2 - 3x - 10 = (x - 5)(x + 2)$ (M1)(A1) (C2)

(b) $x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0$ (M1)

$\Rightarrow x = 5$ or $x = -2$ (A1)(C2)

[4]

2. (a) $p = -\frac{1}{2}$, $q = 2$ (A1)(A1) (C2)

or vice versa

(b) By symmetry C is midway between p, q (M1)

Note: *This (M1) may be gained by implication.*

$\Rightarrow$ x-coordinate is $\dfrac{-\frac{1}{2}+2}{2} = \dfrac{3}{4}$ (A1)(C2)

[4]

3. $(7 - x)(1 + x) = 0$ (M1)

$\Leftrightarrow x = 7$ or $x = -1$ (A1)(C1)(C1)

B: $x = \dfrac{7 + -1}{2} = 3$; (A1)

$y = (7 - 3)(1 + 3) = 16$ (A1)(C2)

[4]

4. $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0$ (M1)

$16k^2 - 4(4)(9) = 0$ (A1)

$k^2 = 9$

$k = \pm 3$ (A1)

But given $k > 0$, $k = 3$ (A1)(C4)

OR

One solution $\Rightarrow (4x^2 + 4kx + 9)$ is a perfect square (M1)

$4x^2 + 4kx + 9 = (2x \pm 3)^2$ by inspection (A2)

given $k > 0$, $k = 3$ (A1)(C4)

[4]

5. Graph of quadratic function.

Expression	+	–	0	
a		✓		(A1) (C1)
c		✓		(A1) (C1)
$b^2 - 4ac$			✓	(A1) (C1)
b	✓			(A1) (C1)

[4]

6. (a) $f(x) = x^2 - 6x + 14$

$f(x) = x^2 - 6x + 9 - 9 + 14$ (M1)

$f(x) = (x - 3)^2 + 5$ (M1)

(b) Vertex is (3, 5) (A1)(A1)

[4]

7. (a) When $t = 0$, (M1)

$h = 2 + 20 \times 0 - 5 \times 0^2 = 2$ $h = 2$ (A1) 2

(b) When $t = 1$, (M1)

$h = 2 + 20 \times 1 - 5 \times 1^2$ (A1)

$= 17$ (AG) 2

(c) (i) $h = 17 \Rightarrow 17 = 2 + 20t - 5t^2$ (M1)

(ii) $5t^2 - 20t + 15 = 0$ (M1)

$\Leftrightarrow 5(t^2 - 4t + 3) = 0$

$\Leftrightarrow (t - 3)(t - 1) = 0$ (M1)

Note: *Award (M1) for factorizing or using the formula*

$\Leftrightarrow t = 3$ or 1 (A1) 4

Note: *Award (A1) for t = 3*

(d) (i) $h = 2 + 20t - 5t^2$

$\Rightarrow \frac{dh}{dt} = 0 + 20 - 10t$

$= 20 - 10t$ (A1)(A1)

(ii) $t = 0$ (M0)

$\Rightarrow \frac{dh}{dt} = 20 - 10 \times 0 = 20$ (A1)

(iii) $\frac{dh}{dt} = 0$ (M1)

$\Leftrightarrow 20 - 10t = 0 \Leftrightarrow t = 2$ (A1)

(iv) $t = 2$ (M1)

$\Rightarrow h = 2 + 20 \times 2 - 5 \times 2^2 = 22 \Rightarrow h = 22$ (A1) 7

[15]

8. $y = (x + 2)(x - 3)$ (M1)

$= x^2 - x - 6$ (A1)

Therefore, $0 = 4 - 2p + q$ (A1)(A1) (C2)(C2)

OR

$y = x^2 - x - 6$ (C3)

OR

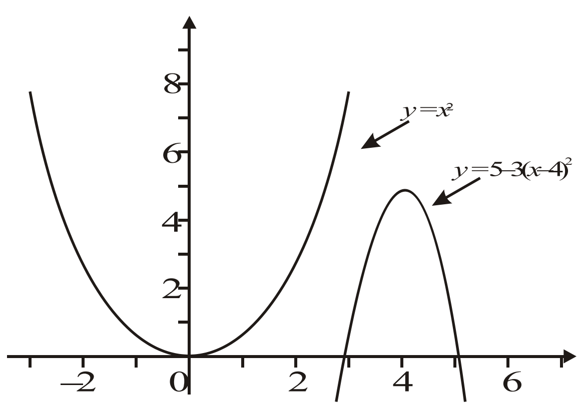
$0 = 4 - 2p + q$ (A1)

$0 = 9 + 3p + q$ (A1)

$p = -1, q = -6$ (A1)(A1) (C2)(C2)

[4]

9.



$q = 5$ (A1)(C1)

$k = 3, p = 4$ (A3)(C3)

[4]

10. (a) (i) $h = -1$ (A2)(C2)

(ii) $k = 2$ (A1)(C1)

(b) $a(1 + 1)^2 + 2 = 0$ (M1)(A1)

$a = -0.5$ (A1)(C3)

[6]

11. (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$ (M1)

$= 2(x - 2)^2 - 3$ (A1)(A1)(A1)

$\Rightarrow a = 2, p = 2, q = -3$ (C4)

(b) Minimum value of $2(x - 2)^2 = 0$ (or minimum value occurs when $x = 2$) (Ml)

$\Rightarrow$ Minimum value of $f(x) = -3$ (A1)(C2)

OR

Minimum value occurs at $(2, -3)$ (M1)(A1) (C2)

[6]

12. **Method 1**

$b^2 - 4ac = 9 - 4k$ (M1)

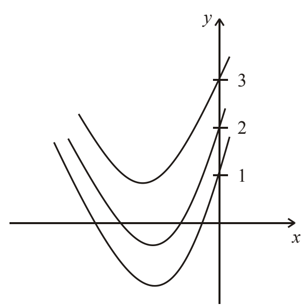
$9 - 4k > 0$ (M1)

$2.25 > k$ (A1)

crosses the x-axis if $k = 1$ **or** $k = 2$ (A1)(A1)

probability $= \frac{2}{7}$ (A1)(C6)

Method 2



(M2)(M1)

Note: *Award (M2) for one (relevant) curve; (M1) for a second one.*

$k = 1$ **or** $k = 2$ (G1)(G1)

probability $= \frac{2}{7}$ (A1)(C6)

[6]

13. (a) Since the vertex is at $(3, 1)$

$h = 3$ (A1)

$k = 1$ (A1) 2

(b) $(5, 9)$ is on the graph $\Rightarrow 9 = a(5 - 3)^2 + 1$ (M1)

$= 4a + 1$ (A1)

$\Rightarrow 9 - 1 = 4a = 8$ (A1)

$\Rightarrow a = 2$ (AG) 3

Note: *Award (M1)(A1)(A0) for using a reverse proof, ie substituting for a, h, k and showing that (5, 9) is on the graph.*

(c) $y = 2(x - 3)^2 + 1$ (M1)

$= 2x^2 - 12x + 19$ (AG) 1

(d) (i) Graph has equation $y = 2x^2 - 12x + 19$

$\frac{dy}{dx} = 4x - 12$ (A1)

(ii) At point (5, 9), gradient = 4(5) – 12 = 8 (A1)

(iii) Equation: $y - 9 = 8(x - 5)$ (M1)(A1)

$8x - y - 31 = 0$

OR

$9 = 8(5) + c$ (M1)

$c = -31$

$y = 8x - 31$ (A1) 4

[10]

14. One solution ⇒ discriminant = 0 (M2)

$3^2 - 4k = 0$ (A2)

$9 = 4k$

$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ (A2)(C6)

Note: *If candidates correctly solve an incorrect equation, award M2 A0 A2(ft), if they have the first line or equivalent, otherwise award no marks.*

[6]

15. (a) $a = 3, b = 4$ (A1)

$f(x) = (x - 3)^2 + 4$ A1 (C2)

(b) $y = (x - 3)^2 + 4$

METHOD 1

$x = (y - 3)^2 + 4$ (M1)

$x - 4 = (y - 3)^2$

$\sqrt{x-4} = y - 3$ (M1)

$y = \sqrt{x-4} + 3$ (A1) 3

METHOD 2

$y - 4 = (x - 3)^2$ (M1)

$\sqrt{y-4} = x - 3$ (M1)

$\sqrt{y-4} + 3 = x$

$y = \sqrt{x-4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x-4} + 3$ (A1) 3

(c) $x \geq 4$ (A1)(C1)

[6]