Functions – Practice Problems

1. The function f is given by $f(x) = \sqrt{\ln(x-2)}$ Find the domain of the function.

Working:

Answer:
.....

(Total 4 marks)

2. Let $f(x) = \sqrt{x+4}$, $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

(a) Find $(g \circ f)$ (3).

(b) Find $f^{-1}(x)$.

(c) Write down the domain of f^{-1}.

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(Total 6 marks)

3. Two functions f and g are defined as follows:

$f(x) = \cos x$, $0 \leq x \leq 2\pi$;

$g(x) = 2x + 1$, $x \in \mathbb{R}$.

Solve the equation $(g \circ f)(x) = 0$.

Working:

Answer:
.....

(Total 4 marks)

4. Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$. Solve the equation

$(f^{-1} \circ g)(x) = 0.25$.

Working:

Answer:

.....

(Total 4 marks)

5. Two functions f, g are defined as follows:

$f : x \to 3x + 5$

$g : x \to 2(1 - x)$

Find

(a) $f^{-1}(2)$;

(b) $(g \circ f)(-4)$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

6. Let $f(x) = 2^x$, and $g(x) = \frac{x}{x-2}$, $(x \neq 2)$.

Find

(a) $(g \circ f)(3)$;

(b) $g^{-1}(5)$.

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

7. Consider the functions $f : x \mapsto 4(x-1)$ and $g : x \mapsto \frac{6-x}{2}$.

(a) Find g^{-1}.

(b) Solve the equation $(f \circ g^{-1})(x) = 4$.

Working:

Answers:
(a) .....
(b) .....

(Total 6 marks)

8. Let $f(x) = e^{-x}$, and $g(x) = \frac{x}{1+x}$, $x \neq -1$. Find

(a) $f^{-1}(x)$;

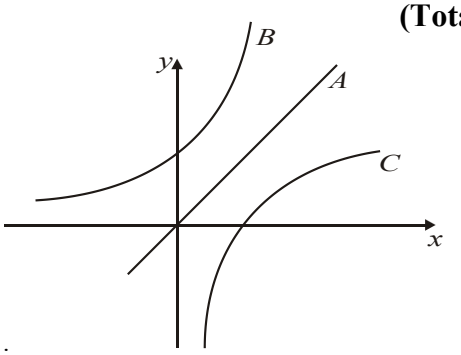
(b) $(g \circ f)(x)$.

Working:

Answers:
(a) .....
(b) .....

(Total 6 marks)

9. The diagram shows three graphs.



A is part of the graph of $y = x$.
B is part of the graph of $y = 2^x$.
C is the reflection of graph *B* in line *A*.

Write down

(a) the equation of *C* in the form $y = f(x)$;

(b) the coordinates of the point where *C* cuts the *x*-axis.

Working:

Answers:
(a) .....
(b) .....

(Total 4 marks)

10. The function f is defined by

$$f(x) : x \mapsto \sqrt{3-2x} \qquad x \leq \frac{3}{2}$$

Evaluate $f^{-1}(5)$.

Working:

Answer:

.....

(Total 4 marks)

11. Given that $f(x) = 2e^{3x}$, find the inverse function $f^{-1}(x)$.

Working:

Answer:

.....

(Total 4 marks)

Functions – Practice Problems – MarkScheme

1. $\ln(x-2) \geq 0$ since we need to find its square root (M1)(R1)
$\Rightarrow x - 2 \geq 1$ (A1)
$\Rightarrow x \geq 3$ (A1) (C4)

Note: *$x > 3$: deduct [1 mark] ([2 marks] if no working shown).*

[4]

2. (a) **METHOD 1**
$f(3) = \sqrt{7}$ (A1)
$(g \circ f)(3) = 7$ A1 N2
METHOD 2
$(g \circ f)(x) = \sqrt{x+4}$ $(= x + 4)$ (A1)
$(g \circ f)(3) = 7$ A1 N2

(b) For interchanging x and y (seen anywhere) (M1)
Evidence of correct manipulation A1
Eg $x = \sqrt{y+4}$, $x^2 = y + 4$
$f^{-1}(x) = x^2 - 4$ A1 N2

(c) $x \geq 0$ A1 N1

[6]

3. $(g \circ f)(x) = 0 \Rightarrow 2\cos x + 1 = 0$ (M1)
$\Rightarrow \cos x = -\frac{1}{2}$ (A1)
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ (A1)(A1)
(C4)

Note: *Accept 120°, 240°.*

[4]

4. $x = g^{-1}(f(0.25))$ (M1)
$= \log_2((0.25)^{1/2})$ (A1)
$= \log_2\left(\frac{1}{2}\right)$ (A1)
$= -1$ (A1)
OR
$f^{-1}(x) = x^2$ (M1)
$= (f^{-1} \circ g)(x) = f^{-1}(2^x) = 2^{2x}$ (M1)
Therefore, $2^{2x} = 0.25 = 2^{-2}$ (M1)
$\Rightarrow 2x = -2$
$\Rightarrow x = -1$ (A1) (C4)

[4]

5. (a) $f^{-1}(2) \Rightarrow 3x + 5 = 2$ (M1)
$x = -1$ (A1)(C2)

(b) $g(f(-4) = g(-12 + 5)$
$= g(-7)$ (A1)
$= 2(1 + 7)$
$= 16$ (A1) (C2)

[4]

6. (a) $f(3) = 2^3$ (M1)

$(g \circ f)(3) = \dfrac{2^3}{2^3 - 2}$ (M1)

$= \dfrac{8}{6}$ (A1)

$(g \circ f)(3) = \dfrac{4}{3}$ (C3)

(b) $x = \dfrac{y}{y-2}$ (M1)

$x(y-2) = y \Rightarrow y(x-1) = 2x$

$\Rightarrow y = \dfrac{2x}{(x-1)}$ (A1)

$y = \dfrac{10}{(5-1)} = 2.5$ (A1) (C3)

Note: *Interchanging x and y may take place at any time.*

[6]

7. (a) $y = \dfrac{6-x}{2}$

$\Rightarrow x = \dfrac{6-y}{2}$ (M1)

$\Rightarrow y = 6 - 2x = g^{-1}(x)$ (A1)(C2)

(b) $(f \circ g^{-1})(x) = 4[(6-2x)-1] = 4(5-2x) = 20 - 8x$ (M1)(A1)

$20 - 8x = 4 \Rightarrow 8x = 16$ (M1)

$\Rightarrow x = 2$ (A1) (C4)

[6]

8. (a) $x = e^{-y}$ (M1)

$\ln x = -y$ (A1)

$y = f^{-1}(x) = -\ln x$ (A1)(C3)

(b) $(g \circ f)(x) = g(e^{-x})$ (M1)

$= \dfrac{e^{-x}}{1+e^{-x}}$ (A2) (C3)

Note: *Award (M1)(A1) for* $= e^{-\frac{x}{1+x}}$ *(ie for* $(f \circ g)(x)$*)*

[6]

9. (a) C has equation $x = 2^y$ (A1)

ie $y = \log_2 x$ (A1)(C2)

OR Equation of B is $x = \log_2 y$ (A1)

Therefore equation of C is $y = \log_2 x$ (A1) (C2)

(b) Cuts x-axis $\Rightarrow \log_2 x = 0$

$x = 2^\circ$ (A1)

$x = 1$

Point is (1, 0) (A1) (C2)

[4]

10. $\sqrt{3-2x}=5$ (M1)

$3-2x=25$ (A1)

$-2x=22$ (A1)

$x=-11$ (A1) (C4)

OR

Let $y=\sqrt{3-2x}$

$\Rightarrow y^2=3-2x$ (M1)

$\Rightarrow x=\dfrac{3-y^2}{2}$ (A1)

$\Rightarrow f^{-1}(x)=\dfrac{3-x^2}{2}$

$\Rightarrow f^{-1}(5)=\dfrac{3-25}{2}$ (M1)

$=-11$ (A1) (C4)

[4]

11. $f(x)=2e^{3x}$. Let $x=2e^{3y}$ (M1)

$\Rightarrow \dfrac{x}{2}=e^{3y}$ (A1)

$\Rightarrow \ln\left(\dfrac{x}{2}\right)=3y$ (A1)

$\Rightarrow y=\dfrac{1}{3}\ln\left(\dfrac{x}{2}\right)$ (A1)

that is $f^{-1}(x)=\dfrac{1}{3}\ln\left(\dfrac{x}{2}\right)$ (C4)

[4]