Exponents & Logs – Practice Problems

1. Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

Working:

Answer:
....

(Total 4 marks)

2. Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

Working:

Answer:
....

(Total 4 marks)

3. If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y, expressions for

(a) $\log_2 5$;

(b) $\log_a 20$.

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

4. Let $\log_{10}P = x$, $\log_{10}Q = y$ and $\log_{10}R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x , y and z.

Working:

Answer:
....

(Total 4 marks)

5. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

Working:

Answer:
....

(Total 4 marks)

6. Consider the following statements

A: $\log_{10}(10^x) > 0$.

B: $-0.5 \le \cos(0.5x) \le 0.5$.

C: $-\frac{\pi}{2} \le \arctan x \le \frac{\pi}{2}$.

(a) Determine which statements are true for all real numbers x. Write your answers (yes or no) in the table below.

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example
A		
B		
C		

(b) If a statement is not true for all x, complete the last column by giving an example of one value of x for which the statement is false.

Working:

(Total 6 marks)

7. Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$

(b) $\log_5\left(\frac{1}{x}\right)$

(c) $\log_{25} x$

Working:

Answers:
(a) ....
(b) ....
(c) ....

(Total 6 marks)

8. A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

Working:

Answer:
....

(Total 4 marks)

9. Initially a tank contains 10 000 litres of liquid. At the time $t = 0$ minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by

$$V = 10\,000\,(0.933^t).$$

(a) Find the value of V after 5 minutes.

(1)

(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank.

(3)

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen.

(3)

(d) (i) Find the value of $10\,000 - V$ when $t = 0.001$ minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures.

(3)

(Total 10 marks)

10. A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\,e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

11. Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

(a) What was the population one year ago?

(b) What was the population five years ago?

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

12. Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc.

(3)

(b) How many complete years will it take for Michele's initial investment to double in value?

(3)

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years?

(4)

(Total 10 marks)

13. Solve the equation $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

Working:

Answer:

....

(Total 6 marks)

14. Consider functions of the form $y = e^{-kx}$

(a) Show that $\int_0^1 e^{-kx}dx = \frac{1}{k}\left(1 - e^{-k}\right)$.

(3)

(b) Let $k = 0.5$

(i) Sketch the graph of $y = e^{-0.5x}$, for $-1 \le x \le 3$, indicating the coordinates of the y-intercept.

(ii) Shade the region enclosed by this graph, the x-axis, y-axis and the line $x = 1$.

(iii) Find the area of this region.

(5)

(c) (i) Find $\frac{dy}{dx}$ in terms of k, where $y = e^{-kx}$.

The point P(1, 0.8) lies on the graph of the function $y = e^{-kx}$.

(ii) Find the value of k in this case.

(iii) Find the gradient of the tangent to the curve at P.

(5)

(Total 13 marks)

15. \$1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed \$3000.

Working:

Answer:

....

(Total 6 marks)

16. The mass m kg of a radio-active substance at time t hours is given by

$$m = 4e^{-0.2t}.$$

(a) Write down the initial mass.

(b) The mass is reduced to 1.5 kg. How long does this take?

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

Exponents & Logs – Practice Problems - Markscheme

1. $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

$3^{2x-2} = 3^{-2x}$ (M1) (A1)

$2x - 2 = -2x$ (A1)

$x = \frac{1}{2}$ (A1) (C4)

[4]

2. $4^{3x-1} = 1.5625 \times 10^{-2}$

$(3x - 1)\log_{10} 4 = \log_{10} 1.5625 - 2$ (M1)

$\Rightarrow 3x - 1 = \frac{\log_{10} 1.5625 - 2}{\log_{10} 4}$ (A1)

$\Rightarrow 3x - 1 = -3$ (A1)

$\Rightarrow x = -\frac{2}{3}$ (A1) (C4)

[4]

3. (a) $\log_2 5 = \frac{\log_a 5}{\log_a 2}$ (M1)

$= \frac{y}{x}$ (A1) (C2)

(b) $\log_a 20 = \log_a 4 + \log_a 5$ or $\log_a 2 + \log_a 10$ (M1)

$= 2\log_a 2 + \log_a 5$

$= 2x + y$ (A1) (C2)

[4]

4. $\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$ (M1)

$2\log_{10}\left(\frac{P}{QR^3}\right) = 2(\log_{10}P - \log_{10}(QR^3))$ (M1)

$= 2(1\text{og}_{10}P - \log_{10}Q - 3\log_{10}R)$ (M1)

$= 2(x - y - 3z)$

$= 2x - 2y - 6z$ or $2(x - y - 3z)$ (A1)

[4]

5. **METHOD 1**

$\log_9 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = 2 - 1 + \frac{1}{2}$ (M1)

$\Rightarrow \frac{3}{2} = \log_9 x$ (A1)

$\Rightarrow x = 9^{\frac{3}{2}}$ (M1)

$\Rightarrow x = 27$ (A1) (C4)

METHOD 2

$\log 81 + \log_9\left(\frac{1}{9}\right) + \log_9 3 = \log_9\left[8 1\left(\frac{1}{9}\right)3\right]$ (M2)

$= \log_9 27$ (A1)

$\Rightarrow x = 27$ (A1) (C4)

[4]

6.

Statement	(a) Is the statement true for all real numbers x? (Yes/No)	(b) If not true, example	
A	No	$x = -1$ $(\log_{10} 0.1 = -1)$	(a) (A3) (C3)
B	No	$x = 0$ $(\cos 0 = 1)$	(b) (A3) (C3)
C	Yes	N/A	

***Notes**: (a) Award (A1) for each correct answer.*
(b) Award (A) marks for statements A and B only if NO in column (a).
Award (A2) for a correct counter example to statement A, (A1) for a correct counter example to statement B (ignore other incorrect examples).
Special Case for statement C:
Award (A1) if candidates write NO, and give a valid reason (eg arctan 1 = $\frac{5\pi}{4}$).

[6]

7. (a) $\log_5 x^2 = 2\log_5 x$ (M1)
$= 2y$ (A1) (C2)

(b) $\log_5 \frac{1}{x} = -\log_5 x$ (M1)
$= -y$ (A1) (C2)

(c) $\log_{25} x = \frac{\log_5 x}{\log_5 25}$ (M1)
$= \frac{1}{2}y$ (A1) (C2) [6]

8. $1.023^t = 2$ (M1)
$\Rightarrow t = \frac{\ln 2}{\ln 1.023}$ (M1)(A1)
$= 30.48...$
30 minutes (nearest minute) (A1) (C4)

***Note:** Do not accept 31 minutes.*

[4]

9. ***Note:** A reminder that a candidate is penalized only once in this question for not giving answers to 3 sf*

(a) $V(5) = 10000 \times (0.933^5) = 7069.8 \ldots$
$= 7070$ (3 sf) (A1) 1

(b) We want t when $V = 5000$ (M1)
$5000 = 10000 \times (0.933)^t$
$0.5 = 0.933^t$ (A1)
$\frac{\log(0.5)}{\log(0.993)} = t$ or $\left(\frac{\ln(0.5)}{\ln(0.993)}\right)$
$9.9949 = t$
After 10 minutes 0 seconds, to nearest second (or 600 seconds). (A1) 3

(c) $0.05 = 0.933^t$ (M1)
$\frac{\log(0.5)}{\log(0.993)} = t = 43.197$ minutes (M1)(A1)
$\approx 3/4$ hour (AG) 3

(d) (i) $10000 - 10000(0.933)^{0.001} = 0.693$ (A1)

(ii) Initial flow rate = $\frac{dV}{dt}$ where $t = 0$, (M1)

$\frac{dv}{dt} = \frac{0.693}{0.001} = 693$

$= 690$ (2 sf) (A1)

OR

$\frac{dv}{dt} = 690$ (G2) 3

[10]

10. (a) At $t = 2$, $N = 10e^{0.4(2)}$ (M1)

$N = 22.3$ (3 sf)

Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$

$10 = e04^{t}$

$\ln 10 = 0.4t$

$t = \frac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

11. (a) $\frac{15.2}{1.027} = 14.8$ million (M1)(A1) (C2)

(b) $\frac{15.2}{(1.027)^5} = 13.3$ million (M1)(A1)(C2)

OR

$\frac{14.8}{(1.027)^4} = 13.3$ million (M1)(A1)(C2)

[4]

12. (a) Value $= 1500(1.0525)^3$ (M1)

$= 1748.87$ (A1)

$= 1749$ (nearest franc) (A1) 3

(b) $3000 = 1500(1.0525)^t \Rightarrow 2 = 1.0525^t$ (M1)

$t = \frac{\log 2}{\log 1.0525} = 13.546$ (A1)

It takes 14 years. (A1) 3

(c) $3000 = 1500(1 + r)^{10}$ or $2(1 + r)^{10}$ (M1)

$\Rightarrow \sqrt[10]{2} = 1 + r$ or $\log 2 = 10 \log (1 + r)$ (M1)

$\Rightarrow r = \sqrt[10]{2} - 1$ or $r = 10^{\frac{\log 2}{10}} - 1$ (A1)

$r = 0.0718$ [or 7.18%] (A1) 4

[10]

13. $\log_{27}(x(x - 0.4)) = 1$ (M1)(A1)

$x^2 - 0.4x = 27$ (M1)

$x = 5.4$ or $x = -5$ (G2)

$x = 5.4$ (A1) (C6)

Note: *Award (C5) for giving both roots.*

[6]

14. (a) $\int_0^1 e^{-kx}dx = \left[-\frac{1}{k}e^{-kx}\right]_0^1$ (A1)

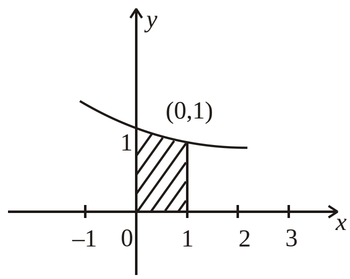
$= -\frac{1}{k}\left(e^{-k} - e^{0}\right)$ (A1)

$= -\frac{1}{k}\left(e^{-k} - 1\right)$ (A1)

$= \frac{1}{k}\left(1 - e^{-k}\right)$ (AG) 3

(b) $k = 0.5$

(i)



(A2)

Note: *Award (A1) for shape, and (A1) for the point (0,1).*

(ii) Shading (see graph) (A1)

(iii) Area = $\int_0^1 e^{-kx}dx$ for k = 0.5 (M1)

$= \frac{1}{0.5}(1 - e^{0.5})$

= 0.787 (3 sf) (A1)

OR

Area = 0.787 (3 sf) (G2) 5

(c) (i) $\frac{dy}{dx} = -ke^{-kx}$ (A1)

(ii) $x = 1$ $y = 0.8$ $\Rightarrow 0.8 = e^{-k}$ (A1)

$\ln 0.8 = -k$

$k = 0.223$ (A1)

(iii) At $x = 1$ $\frac{dy}{dx} = -0.223e^{-0.223}$ (M1)

$= -0.179$ (accept -0.178) (A1)

OR

$\frac{dy}{dx} = -0.178$ or -0.179 (G2) 5

[13]

15. 15% per annum = $\frac{15}{12}$% = 1.25% per month (M1)(A1)

Total value of investment after n months, $1000(1.0125)^n > 3000$ (M1)

=> $(1.0125)^n > 3$

$n \log(1.0125) > \log(3)$ => $n > \frac{\log 3}{\log 1.0125}$ (M1)

Whole number of months required so $n = 89$ months. (A1) (C6)

Notes: *Award (C5) for the answer of 90 months obtained from using n – 1 instead of n to set up the equation.*
Award (C2) for the answer 161 months obtained by using simple interest.
Award (C1) for the answer 160 months obtained by using simple interest.

[6]

16. (a) Initial mass $\Rightarrow t = 0$ (A1)
mass = 4 (A1) (C2)

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) (M2)
$\ln 0.375 = -0.2t$ (M1)
$t = 4.90$ hours (A1) (C4)