## Series – Practice Problems

**1.** Find the sum of the arithmetic series

$$17 + 27 + 37 + ... + 417.$$

*Working:*

*Answer*: ..........................................................................

**(Total 4 marks)**

**2.** An arithmetic series has five terms. The first term is 2 and the last term is 32. Find the sum of the series.

*Working:*

*Answer:* ..........................................................................

**(Total 4 marks)**

**3.** In an arithmetic sequence, the first term is 5 and the fourth term is 40. Find the second term.

*Working:*

*Answer:* ..........................................................................

**(Total 4 marks)**

**4.** Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.

(a) On which day does she run a distance of 10 km in training?

(b) What is the total distance she will have run in training by the end of that day? Give your answer exactly.

*Working:*

*Answers*:
(a) ..................................................................
(b) ..................................................................

**(Total 4 marks)**

**5.** The first three terms of an arithmetic sequence are 7, 9.5, 12.

(a) What is the $41^{st}$ term of the sequence?

(b) What is the sum of the first 101 terms of the sequence?

*Working:*

*Answers*:
(a) ..................................................................
(b) ..................................................................

**(Total 4 marks)**

**6.** In an arithmetic sequence, the first term is –2, the fourth term is 16, and the $n^{\text{th}}$ term is 11 998.
(a) Find the common difference $d$.
(b) Find the value of $n$.

*Working:*

*Answers*:
(a) ..................................................................
(b) ..................................................................

**(Total 6 marks)**

**7.** Find the sum of the infinite geometric series $\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{18}+\ldots$

*Working:*

*Answer:*
......................................................................

**(Total 4 marks)**

**8.** \$1000 is invested at the beginning of each year for 10 years.
The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.
Calculate, giving your answers to the nearest dollar
(a) how much the first \$1000 is worth at the end of the ten years;
(b) the total value of the investments at the end of the ten years.

*Working:*

*Answers*:
(a) ..................................................................
(b) ..................................................................

**(Total 4 marks)**

**9.** Portable telephones are first sold in the country *Cellmania* in 1990. During 1990, the number of units sold is 160. In 1991, the number of units sold is 240 and in 1992, the number of units sold is 360.
In 1993 it was noticed that the annual sales formed a geometric sequence with first term 160, the 2nd and 3rd terms being 240 and 360 respectively.
(a) What is the common ratio of this sequence?
**(1)**

Assume that this trend in sales continues.
(b) How many units will be sold during 2002?
**(3)**

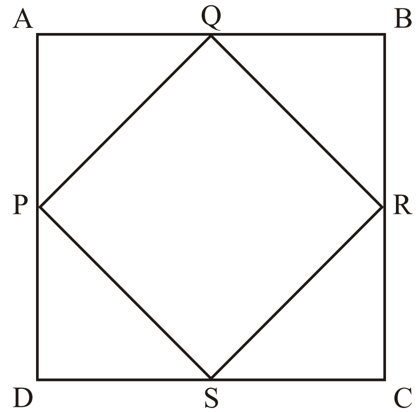
(c) In what year does the number of units sold first exceed 5000?
**(4)**

Between 1990 and 1992, the total number of units sold is 760.
(d) What is the total number of units sold between 1990 and 2002?
**(2)**

During this period, the total population of *Cellmania* remains approximately 80 000.
(e) Use this information to suggest a reason why the geometric growth in sales would not continue.
**(1)**

**(Total 11 marks)**

**10.** The diagram shows a square ABCD of side 4 cm. The midpoints P, Q, R, S of the sides are joined to form a **second** square.

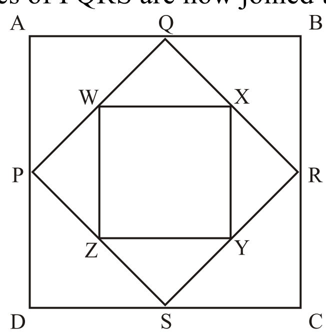



(a) (i) Show that PQ = $2\sqrt{2}$ cm.

(ii) Find the area of PQRS.

**(3)**

The midpoints W, X, Y, Z of the sides of PQRS are now joined to form a **third** square as shown.




(b) (i) Write down the area of the **third** square, WXYZ.

(ii) Show that the areas of ABCD, PQRS, and WXYZ form a geometric sequence. Find the common ratio of this sequence.

**(3)**

The process of forming smaller and smaller squares (by joining the midpoints) is **continued indefinitely**.

(c) (i) Find the area of the $11^{\text{th}}$ square.

(ii) Calculate the sum of the areas of **all** the squares.

**(4)**

**(Total 10 marks)**

**11.** Gwendolyn added the multiples of 3, from 3 to 3750 and found that

$$3 + 6 + 9 + \ldots + 3750 = s.$$

Calculate $s$.

*Working:*

*Answer*:

..................................................................

**(Total 6 marks)**

**12.** The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.

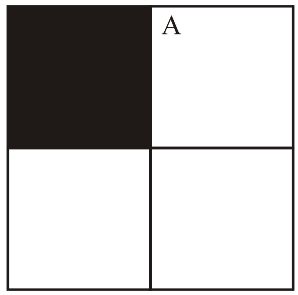



Diagram 1

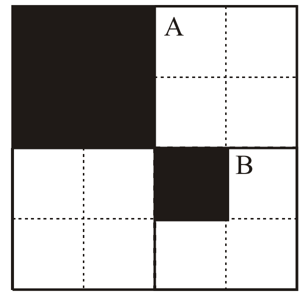



Diagram 2

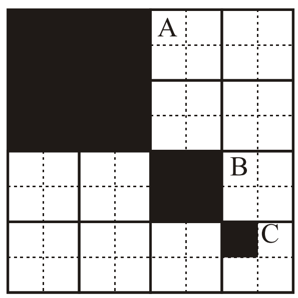



Diagram 3

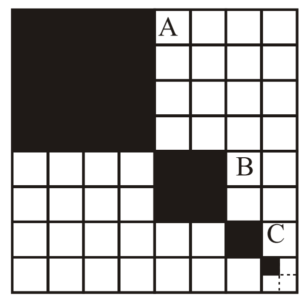



Diagram 4

(a) (i) Find the area of square B and of square C.

(ii) Show that the areas of squares A, B and C are in geometric progression.

(iii) Write down the common ratio of the progression.

**(5)**

(b) (i) Find the **total** area shaded in diagram 2.

(ii) Find the **total** area shaded in the 8$^{th}$ diagram of this sequence. Give your answer correct to six significant figures.

**(4)**

(c) The dividing and shading process illustrated is continued indefinitely. Find the total area shaded.

**(2)**

**(Total 11 marks)**

## Series – Practice Problems - Markscheme

**1.** $17 + 27 + 37 + ... + 417$

$17 + (n - 1)10 = 417$ (M1)

$10(n - 1) = 400$

$n = 41$ (A1)

$S_{41} = \frac{41}{2}(2(17) + 40(10))$ (M1)

$= 41(17 + 200)$

$= 8897$ (A1)

**OR**

$S_{41} = \frac{41}{2}(17 + 417)$ (M1)

$= \frac{41}{2}(434)$

$= 8897$ (A1) (C4)

**[4]**

**2.** $S_5 = \frac{5}{2}\{2 + 32\}$ (M1)(A1)(A1)

$S_5 = 85$ (A1)

**OR**

$a = 2, a + 4d = 32$ (M1)

$\Rightarrow 4d = 30$

$d = 7.5$ (A1)

$S_5 = \frac{5}{2}(4 + 4(7.5))$ (M1)

$= \frac{5}{2}(4 + 30)$

$S_5 = 85$ (A1) (C4)

**[4]**

**3.** $a = 5$

$a + 3d = 40$ (may be implied) (M1)

$d = \frac{35}{3}$ (A1)

$T_2 = 5 + \frac{35}{3}$ (A1)

$= 16\frac{2}{3}$ or $\frac{50}{3}$ or 16.7 (3 sf) $\frac{10\,000 - 1000}{250}$ (A1) (C4)

**[4]**

**4.** (a) $a_1 = 1000$, $a_n = 1000 + (n - 1)250 = 10000$ (M1)

$n = \frac{10000 - 1000}{250} + 1 = 37$.

She runs 10 km on the 37th day. (A1)

(b) $S_{37} = \frac{37}{2}(1000 + 10000)$ (M1)

She has run a total of 203.5 km (A1)

**[4]**

**5.** (a) $u_1 = 7, d = 2.5$ (M1)

$u_{41} = u_1 + (n-1)d = 7 + (41-1)2.5$

$= 107$ (A1)(C2)

(b) $S_{101} = \frac{n}{2}[2u_1 + (n-1)d]$

$= \frac{101}{2}[2(7) + (101-1)2.5]$ (M1)

$= \frac{101(264)}{2}$

$= 13332$ (A1) (C2)

**[4]**

**6.** (a) $u_4 = u_1 + 3d$ or $16 = -2 + 3d$ (M1)

$d = \frac{16-(-2)}{3}$ (M1)

$= 6$ (A1)(C3)

(b) $u_n = u_1 + (n-1)6$ or $11998 = -2 + (n-1)6$ (M1)

$n = \frac{11998+2}{6} + 1$ (A1)

$= 2001$ (A1) (C3)

**[6]**

**7.** $S = \frac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ (M1)(A1)

$= \frac{2}{3} \times \frac{3}{5}$ (A1)

$= \frac{2}{5}$ (A1) (C4)

**[4]**

**8.** (a) $\$1000 \times 1.075^{10} = \$2061$ (nearest dollar) (A1)(C1)

(b) $1000(1.075^{10} + 1.075^{9} + ... + 1.075)$ (M1)

$= \frac{1000(1.075)(1.075^{10}-1)}{1.075-1}$ (M1)

$= \$15208$ (nearest dollar) (A1) (C3)

**[4]**

**9.** (a) $r = \frac{360}{240} = \frac{240}{160} = \frac{3}{2} = 1.5$ (A1) 1

(b) 2002 is the 13th year. (M1)

$u_{13} = 160(1.5)^{13-1}$ (M1)

$= 20759$ (Accept 20760 or 20800.) (A1) 3

(c) $5000 = 160(1.5)^{n-1}$

$\frac{5000}{160} = (1.5)^{n-1}$ (M1)

$\log\left(\frac{5000}{160}\right) = (n-1)\log 1.5$ (M1)

$n - 1 = \frac{\log\left(\frac{5000}{160}\right)}{\log 1.5} = 8.49$ (A1)

$\Rightarrow n = 9.49 \Rightarrow 10^{\text{th}}$ year

$\Rightarrow 1999$ (A1)

**OR**

Using a gdc with $u_1 = 160$, $u_{k+1} = \frac{3}{2}u_k$, $u_9 = 4100$, $u_{10} = 6150$ (M2)

1999 (G2) 4

(d) $S_{13} = 160\left[\frac{1.5^{13}-1}{1.5-1}\right]$ (M1)

= 61958 (Accept 61960 or 62000.) (A1) 2

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one. (R1)

**OR**

Sales would saturate. (R1) 1

**[11]**

**10.** (a) (i) $PQ = \sqrt{AP^2 + AQ^2}$ (M1)

$= \sqrt{2^2+2^2} = \sqrt{4(2)} = 2\sqrt{2}$ cm (A1)(AG)

(ii) Area of PQRS $= \left(2\sqrt{2}\right)\left(2\sqrt{2}\right) = 8\ \text{cm}^2$ (A1) 3

(b) (i) Side of third square $= \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{4} = 2$ cm

Area of third square = 4 cm$^2$ (A1)

(ii) $\frac{1^{st}}{2^{nd}} = \frac{16}{8}\quad \frac{2^{nd}}{3r^{nd}} = \frac{8}{4}\left(\frac{1}{2}\right)^{10}$ (M1)

$\Rightarrow$ Geometric progression, $r = \frac{8}{16} = \frac{4}{8} = \frac{1}{2}$ (A1) 3

(c) (i) $u_{11} = u_1 r^{10} = 16\left(\frac{1}{2}\right)^{10} = \frac{16}{1024}$ (M1)

$= \frac{1}{64}$ ( = 0.015625 = 0.0156, 3 sf) (A1)

(ii) $S_\infty = \dfrac{u_1}{1-r} = \dfrac{16}{1-\frac{1}{2}}$ (M1)

$= 32$ (A1) 4

**[10]**

**11.** Arithmetic sequence $d = 3$ (may be implied) (M1)(A1)

$n = 1250$ (A2)

$S = \dfrac{1250}{2}(3 + 3750)$ $\left(\text{or } S = \dfrac{1250}{2}\left(6 + 1249 \times 3\right)\right)$ (M1)

$= 2\ 345\ 625$ (A1)(C6)

**[6]**

**12.** (a) (i) Area B $= \dfrac{1}{16}$ , area C $= \dfrac{1}{64}$ (A1)(A1)

(ii) $\dfrac{\frac{1}{16}}{\frac{1}{4}} = \dfrac{1}{4}$ $\dfrac{\frac{1}{64}}{\frac{1}{16}} = \dfrac{1}{4}$ (Ratio is the same.) (M1)(R1)

(iii) Common ratio $= \dfrac{1}{4}$ (A1) 5

(b) (i) Total area ($S_2$) $= \dfrac{1}{4} + \dfrac{1}{16} = \dfrac{5}{16}$ = (= 0.3125) (0.313, 3 sf) (A1)

(ii) Required area $= S_8 = \dfrac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^8\right)}{1-\frac{1}{4}}$ $\dfrac{\frac{1}{4}}{1-\frac{1}{4}}$ (M1)

$= 0.333328\ 2(471...)$ (A1)

$= 0.333328$ (6 sf) (A1) 4

***Note:*** *Accept result of adding together eight areas correctly.*

(c) Sum to infinity $= \dfrac{\frac{1}{4}}{1-\frac{1}{4}}$ (A1)

$= \dfrac{1}{3}$ (A1) 2

**[11]**