Binomial Expansion & Theorem – Practice Problems

1. Find the coefficient of x^5 in the expansion of $(3x - 2)^8$.

Working:

Answer:

....

(Total 4 marks)

2. Find the coefficient of a^3b^4 in the expansion of $(5a + b)^7$.

Working:

Answer:

....

(Total 4 marks)

3. Find the coefficient of a^5b^7 in the expansion of $(a + b)^{12}$.

Working:

Answer:

....

(Total 4 marks)

4. Determine the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$.

Working:

Answer:

....

(Total 4 marks)

5. Use the binomial theorem to complete this expansion.

$$(3x + 2y)^4 = 81x^4 + 216x^3y + \ldots$$

Working:

Answer:

....

(Total 4 marks)

6. Consider the binomial expansion $(1+x)^4 = 1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + x^4$.

(a) By substituting $x = 1$ into both sides, or otherwise, evaluate $\binom{4}{1} + \binom{4}{2} + \binom{4}{3}$.

(b) Evaluate $\binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \binom{9}{5} + \binom{9}{6} + \binom{9}{7} + \binom{9}{8}$.

Working:

Answers:
(a) ....
(b) ....

(Total 4 marks)

7. Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion?
(b) Find the constant term in this expansion.

Working:

Answers:
(a) ....
(b) ....

(Total 6 marks)

8. Find the coefficient of x^3 in the expansion of $(2 - x)^5$.

Working:

Answer:
....

(Total 6 marks)

9. Find the term containing x^{10} in the expansion of $(5 + 2x^2)^7$.

Working:

Answer:
....

(Total 6 marks)

10. Complete the following expansion.

$(2 + ax)^4 = 16 + 32ax + \ldots$

Working:

Answer:
....

(Total 6 marks)

Binomial Expansion & Theorem – Practice Problems - Markscheme

1. Required term is $\binom{8}{5}(3x)^5(-2)^3$ (A1)(A1)(A1)

Therefore the coefficient of x^5 is $56 \times 243 \times -8$

$= -108864$ (A1) (C4)

[4]

2. $(5a + b)^7 = ...+ \binom{7}{4}(5a)^3(b)^4 + ...$ (M1)

$= \dfrac{7\times6\times5\times4}{1\times2\times3\times4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4$ (M1)(A1)

So the coefficient is 4375 (A1) (C4)

[4]

3. $(a + b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5}=\binom{12}{7}$ (M1)(A1)

$= 792$ (A2) (C4)

[4]

4. The constant term will be the term independent of the variable x. (R1)

$\left(x-\dfrac{2}{x^2}\right)^9 = x^9 + 9x^8\left(\dfrac{-2}{x^2}\right)+...+\binom{9}{3}x^6\left(\dfrac{-2}{x^2}\right)^3 +...+\left(\dfrac{-2}{x^2}\right)^9$ (M1)

$\binom{9}{3}x^6\left(\dfrac{-2}{x^2}\right)^3 = 84x^6\left(\dfrac{-8}{x^6}\right)$ (A1)

$= -672$ (A1)

[4]

5. $(3x + 2y)^4 = (3x)^4 + \binom{4}{1}(3x)^2(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 +(2y)^4$ (A1)

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ (A1)(A1)(A1)

(C4)

[4]

6. (a) $(1 + 1)^4 = 2^4 = 1 + \binom{4}{1}(1) + \binom{4}{2}(1^2) + \binom{4}{3}(1^3) + 1^4$ (M1)

$\Rightarrow \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 16 - 2$

$= 14$ (A1) (C2)

(b) $(1 + 1)^9 = 1 + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} + 1$ (M1)

$\Rightarrow \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} = 2^9 - 2$

$= 510$ (A1) (C2)

[4]

7. (a) 10 (A2) (C2)

(b) $(3x^2)^3 \left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6} 3^3 x^6 \frac{1}{x^6} \left(\text{or } 84 \times 3^3 x^6 \frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

[6]

8. Term involving x^3 is $\binom{5}{3}(2)^2(-x)^3$ (A1)(A1)(A1)

$\binom{5}{3} = 10$ (A1)

Therefore, term = $-40x^3$ (A1)

$\Rightarrow$ The coefficient is –40 (A1) (C6)

[6]

9. Selecting one term (may be implied) (M1)

$\binom{7}{2} 5^2 (2x^2)^5$ (A1)(A1)(A1)

$= 16800x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16800.*

[6]

10. $\ldots + 6 \times 2^2(ax)^2 + 4 \times 2(ax)^3 + (ax)^4$ (M1)(M1)(M1)

$= \ldots + 24a^2x^2 + 8a^3x^3 + a^4x^4$ (A1)(A1)(A1)

(C6)

Notes: *Award C3 if brackets omitted, leading to* $24ax^2 + 8ax^3 + ax^4$. *Award C4 if correct expression with brackets as in first line of markscheme is given as final answer.*

[6]