Binomial Expansion & Theorem – Practice Problems

1. Find the coefficient of \(x^5\) in the expansion of \((3x - 2)^8\).

   Working:

   Answer: .................................................................
   (Total 4 marks)

2. Find the coefficient of \(a^3 b^4\) in the expansion of \((5a + b)^7\).

   Working:

   Answer: .................................................................
   (Total 4 marks)

3. Find the coefficient of \(a^5 b^7\) in the expansion of \((a + b)^{12}\).

   Working:

   Answer: .................................................................
   (Total 4 marks)

4. Determine the constant term in the expansion of \(\left(\frac{x - 2}{x^2}\right)^9\).

   Working:

   Answer: .................................................................
   (Total 4 marks)

5. Use the binomial theorem to complete this expansion.

   \((3x + 2y)^4 = 81x^4 + 216x^3 y +...

   Working:

   Answer: .................................................................
   (Total 4 marks)
6. Consider the binomial expansion \((1+x)^4 = 1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + x^4\).

(a) By substituting \(x = 1\) into both sides, or otherwise, evaluate \(\binom{4}{1} + \binom{4}{2} + \binom{4}{3}\).

(b) Evaluate \(\binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \binom{9}{5} + \binom{9}{6} + \binom{9}{7} + \binom{9}{8}\).

Working:

Answers:
(a) ...........................................................
(b) ...........................................................

(Total 4 marks)

7. Consider the expansion of \(\left(3x^2 - \frac{1}{x}\right)^9\).

(a) How many terms are there in this expansion?
(b) Find the constant term in this expansion.

Working:

Answers:
(a) ...........................................................
(b) ...........................................................

(Total 6 marks)

8. Find the coefficient of \(x^3\) in the expansion of \((2-x)^5\).

Working:

Answer: ...........................................................

(Total 6 marks)

9. Find the term containing \(x^{10}\) in the expansion of \((5 + 2x^2)^7\).

Working:

Answer: ...........................................................

(Total 6 marks)

10. Complete the following expansion.
\[
(2 + ax)^4 = 16 + 32ax + \ldots
\]

Working:

Answer: ...........................................................

(Total 6 marks)
Binomial Expansion & Theorem – Practice Problems - Markscheme

1. Required term is \( \binom{8}{5}(3x)^5(-2)^3 \)  
   Therefore the coefficient of \( x^5 \) is \( 56 \times 243 \times -8 \)  
   \( = -108864 \)  
   \( \text{(A1)(A1)(A1)} \)

2. \( (5a + b)^7 = \ldots + \left( \begin{array}{c} 7 \\ 4 \end{array} \right)(5a)^3(b)^4 + \ldots \)  
   \( = \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4 \)  
   So the coefficient is 4375  
   \( \text{(A1)(A1)} \)

3. \( (a + b)^{12} \)  
   Coefficient of \( a^5b^7 \) is \( \binom{12}{5} = \left( \begin{array}{c} 12 \\ 7 \end{array} \right) \)  
   \( = 792 \)  
   \( \text{(M1)(A1)} \)

4. The constant term will be the term independent of the variable \( x \).  
   \( \left( x - \frac{2}{x^2} \right)^9 = x^9 + 9x^8\left( -\frac{2}{x^2} \right) + \ldots + \binom{9}{6}x^3\left( -\frac{2}{x^2} \right)^3 + \ldots + \left( -\frac{2}{x^2} \right)^9 \)  
   \( = 84x^6\left( -\frac{8}{x^6} \right) \)  
   \( = -672 \)  
   \( \text{(A1)(A1)} \)

5. \( (3x + 2y)^4 = (3x)^4 + \binom{4}{1}(3x)^3(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4 \)  
   \( = 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4 \)  
   \( \text{(A1)(A1)(A1)} \)

[4]
6. (a) \((1 + 1)^4 = 2^4 = 1 + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + 1^4\)  
\[
\Rightarrow \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 16 - 2 = 14
\]  
(M1)  

(b) \((1 + 1)^9 = 1 + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + ... + \binom{9}{8} + 1\)  
\[
\Rightarrow \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + ... + \binom{9}{8} = 2^9 - 2 = 510
\]  
(A1) (C2)  

7. (a) \(10\)  
(A2) (C2)  
[4]  
(b) \((3x^2)^3 \left( \frac{-1}{x} \right)^6\) \([\text{for correct exponents}]\)  
\[
\left( \frac{9}{6} \right)^3 x^6 \left( \frac{1}{x^6} \right) = \text{constant} = 2268
\]  
(M1)(A1)  
(A1) (C4)  
[6]  

8. Term involving \(x^3\) is \(\binom{5}{3} (2)^2 (-x)^3\)  
\[
\binom{5}{3} = 10
\]  
(A1)  
Therefore, term = \(-40x^3\)  
(A1)  
\[\Rightarrow\] The coefficient is \(-40\)  
(A1) (C6)  
[6]  

9. Selecting one term (may be implied)  
\[
\binom{7}{2} 5^2 (2x^2)^5
\]  
\[= 16800x^{10}\]  
(A1)(A1) (C6)  
[6]  

Note: Award C5 for 16800.  

10. \(... + 6 \times 2^2 (ax)^2 + 4 \times 2(ax)^3 + (ax)^4\)  
\[
= ... + 24a^2 x^2 + 8a^3 x^3 + a^4 x^4
\]  
(M1)(M1)(M1)  
(C6)  

Notes: Award C3 if brackets omitted, leading to \(24ax^2 + 8ax^3 + ax^4\).  
Award C4 if correct expression with brackets as in first line of mark scheme is given as final answer.  
[6]