SL Calculus Practice Problems

1. The point P $\left(\frac{1}{2}, 0\right)$ lies on the graph of the curve of $y = \sin(2x - 1)$.

Find the gradient of the tangent to the curve at P.

Working:

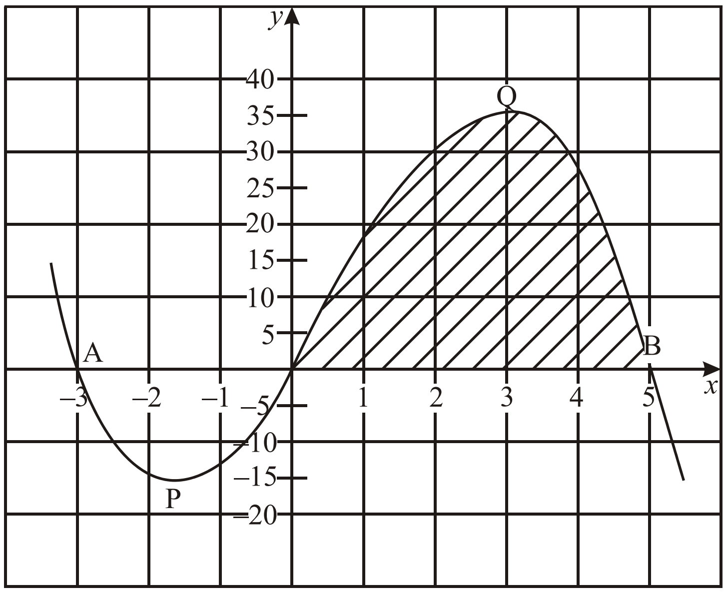
Answer:

....

(Total 4 marks)

2. The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x-axis at A(–3, 0), B(5, 0) and the origin, O. There is a minimum point at P and a maximum point at Q.

(a) The function may also be written in the form $f : x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a;

(ii) b.

(2)

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q.

(7)

(c) (i) Find the equation of the tangent to the graph of f at O.

(ii) This tangent cuts the graph of f at another point. Give the x-coordinate of this point.

(4)

(d) Determine the area of the shaded region.

(2)

(Total 15 marks)

3. A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t}, \ t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.
(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?
(ii) As t becomes large, what value does a approach?
(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

(d) Let y metres be the distance fallen after t seconds.
(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.
(ii) Given that $y = 0$ when $t = 0$, find the value of k.
(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

(7)
(Total 15 marks)

4. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

Answer:
....

(Total 6 marks)

5. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \frac{ds}{dt} = 30 - at$, where a is a constant.

Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

(5)

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.
(i) Write down its velocity as it passes the signal.
(ii) Show that it will stop before reaching the station.

(5)

(c) Train 2 slows down so that it stops at the station. Its velocity is given by
$v = \frac{ds}{dt} = 30 - at$, where a is a constant.
(i) Find, in terms of a, the time taken to stop.
(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

(5)
(Total 15 marks)

6. Consider the function $h(x) = x^{\frac{1}{5}}$.
(i) Find the equation of the tangent to the graph of h at the point where $x = a$, $(a \neq 0)$. Write the equation in the form $y = mx + c$.
(ii) Show that this tangent intersects the x-axis at the point $(-4a, 0)$.

(Total 5 marks)

7. An aircraft lands on a runway. Its velocity v m s^{-1} at time t seconds after landing is given by the equation $v = 50 + 50e^{-0.5t}$, where $0 \le t \le 4$.

(a) Find the velocity of the aircraft
(i) when it lands;
(ii) when $t = 4$.

(4)

(b) Write down an integral which represents the distance travelled in the first four seconds.

(3)

(c) Calculate the distance travelled in the first four seconds.

(2)

After four seconds, the aircraft slows down (decelerates) **at a constant rate** and comes to rest when $t = 11$.

(d) **Sketch** a graph of velocity against time for $0 \le t \le 11$. Clearly label the axes and mark on the graph the point where $t = 4$.

(5)

(e) Find the constant rate at which the aircraft is slowing down (decelerating) between $t = 4$ and $t = 11$.

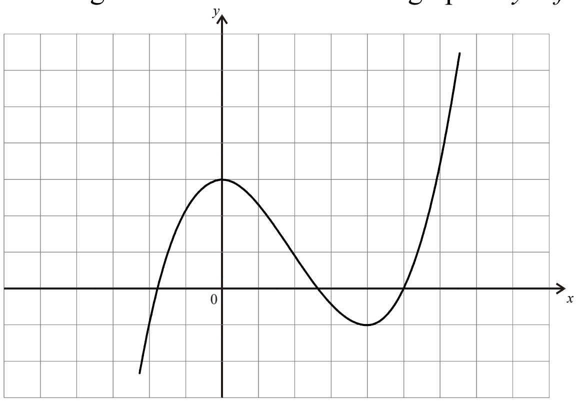
(2)

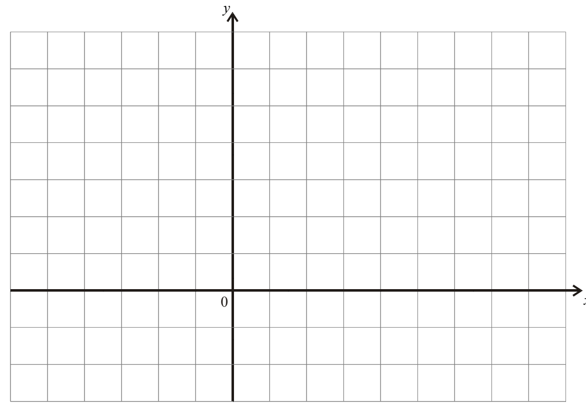
(f) Calculate the distance travelled by the aircraft between $t = 4$ and $t = 11$.

(2)

(Total 18 marks)

8. The diagram on the left shows the graph of $y = f(x)$.





On the right hand grid sketch the graph of $y = f'(x)$.

(Total 6 marks)

9. Consider the function $f(x) = 1 + e^{-2x}$.

(a) (i) Find $f'(x)$.

(ii) Explain briefly how this shows that $f(x)$ is a decreasing function for all values of x (*ie* that $f(x)$ always decreases in value as x increases).

(2)

Let P be the point on the graph of f where $x = -\frac{1}{2}$.

(b) Find an expression in terms of e for

(i) the y-coordinate of P;

(ii) the gradient of the tangent to the curve at P.

(2)

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = ax + b$.

(3)

(d) (i) Sketch the curve of f for $-1 \le x \le 2$.

(ii) Draw the tangent at $x = -\frac{1}{2}$.

(iii) Shade the area enclosed by the curve, the tangent and the y-axis.

(iv) Find this area.

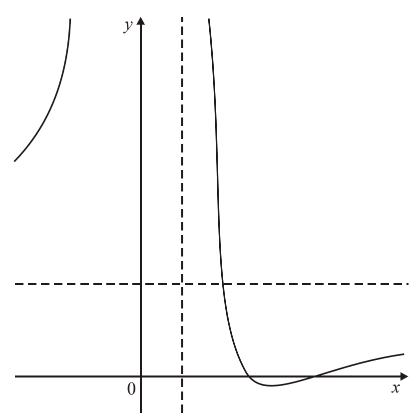
(7)

(Total 14 marks)

10. Consider the function f given by $f(x) = \frac{2x^2 - 13x + 20}{(x-1)^2}$, $\quad x \neq 1$.

A part of the graph of f is given at right.

The graph has a vertical asymptote and a horizontal asymptote, as shown.



(a) Write down the **equation** of the vertical asymptote.

(1)

(b) $f(100) = 1.91 \quad f(-100) = 2.09 \quad f(1000) = 1.99$

(i) Evaluate $f(-1000)$.

(ii) Write down the **equation** of the horizontal asymptote.

(2)

(c) Show that $f'(x) = \frac{9x - 27}{(x-1)^3}$, $\quad x \neq 1$.

(3)

The second derivative is given by $f''(x) = \dfrac{72-18x}{(x-1)^4}$, $x \neq 1$.

(d) Using values of $f'(x)$ and $f''(x)$ explain why a minimum must occur at $x = 3$.

(2)

(e) There is a point of inflexion on the graph of f. Write down the coordinates of this point.

(2)

(Total 10 marks)

11. A car starts by moving from a fixed point A. Its velocity, v m s^{-1} after t seconds is given by $v = 4t + 5 - 5e^{-t}$. Let d be the displacement from A when $t = 4$.

(a) Write down an integral which represents d.

(b) Calculate the value of d.

Working:

Answers:
(a) ……………………………………………..
(b) ……………………………………………..

(Total 6 marks)

12. The displacement s metres of a car, t seconds after leaving a fixed point A, is given by

$$s = 10t - 0.5t^2.$$

(a) Calculate the velocity when $t = 0$.

(b) Calculate the value of t when the velocity is zero.

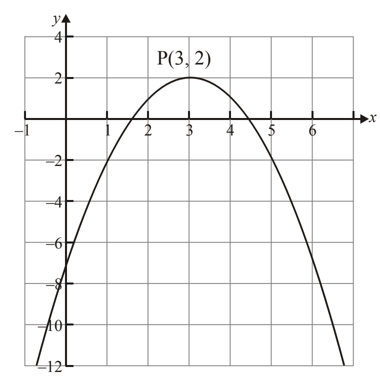
(c) Calculate the displacement of the car from A when the velocity is zero.

Working:

Answers:
(a) ……………………………………………..
(b) ……………………………………………..
(c) ……………………………………………..

(Total 6 marks)

13. The function $f(x)$ is defined as $f(x) = -(x - h)^2 + k$. The diagram at right shows part of the graph of $f(x)$. The maximum point on the curve is P (3, 2).



(a) Write down the value of

(i) h;

(ii) k.

(2)

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

(1)

(c) Find $f'(x)$.

(2)

The point Q lies on the curve and has coordinates (4, 1). A straight line L, through Q, is perpendicular to the tangent at Q.

(d) (i) Calculate the gradient of L.

(ii) Find the equation of L.

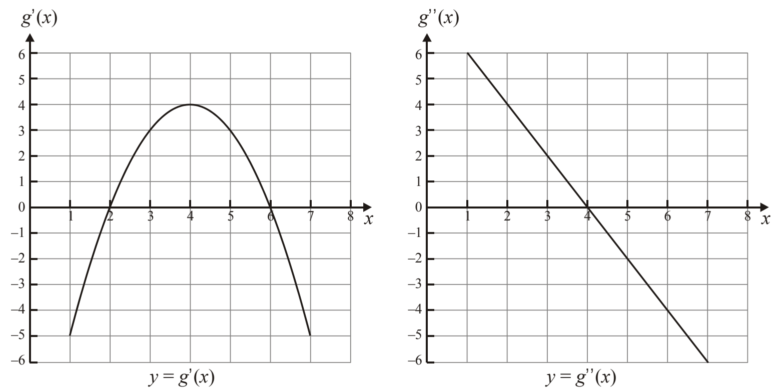
(iii) The line L intersects the curve again at R. Find the x-coordinate of R.

(8)

(Total 13 marks)

14. Let $y = g(x)$ be a function of x for $1 \leq x \leq 7$. The graph of g has an inflexion point at P, and a minimum point at M.

Partial sketches of the curves of g' and g'' are shown below.



Use the above information to answer the following.

(a) Write down the x-coordinate of P, **and** justify your answer.

(2)

(b) Write down the x-coordinate of M, **and** justify your answer.

(2)

(c) Given that $g(4) = 0$, sketch the graph of g. On the sketch, mark the points P and M.

(4)

(Total 8 marks)

15. The velocity v m s^{-1} of a moving body at time t seconds is given by $v = 50 - 10t$.

(a) Find its acceleration in m s^{-2}.

(b) The initial displacement s is 40 metres. Find an expression for s in terms of t.

....

....

....

....

(Total 6 marks)

16. The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down

(i) $f'(x)$;

(ii) $f'(0)$.

(2)

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$.

(3)

Let $g : x \mapsto -0.5x + 2.5$

(c) (i) Find the solutions of $f(x) = g(x)$.

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve.

(6)

(d) Let R be the region enclosed between the curve and N.

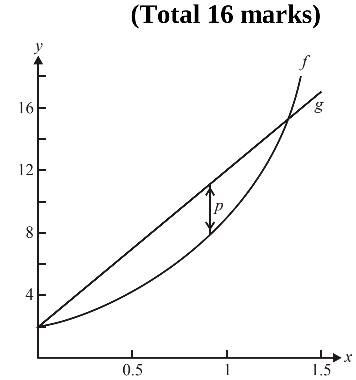
(i) Write down an expression for the area of R.

(ii) Hence write down the area of R.

(5)

(Total 16 marks)

17. The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$.

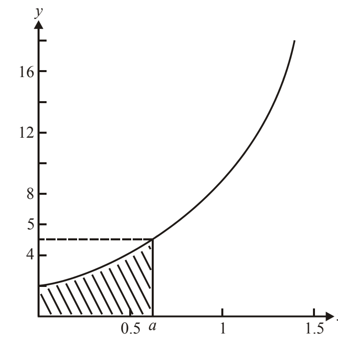


(a) (i) Write down an expression for the vertical distance p between the graphs of f and g.

(ii) Given that p has a maximum value for $0 \le x \le 1.5$, find the value of x at which this occurs.

(6)

The graph of $y = f(x)$ only is shown in the diagram at right. When $x = a$, $y = 5$.



(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$.

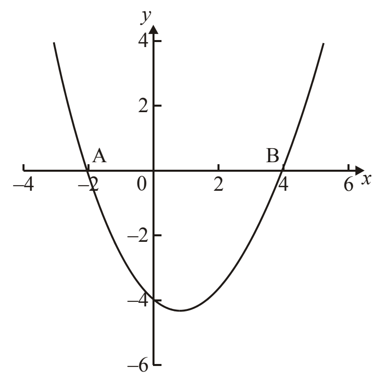
(5)

(c) The region shaded in the diagram is rotated through 360° about the x-axis. Write down an expression for the volume obtained.

(3)

(Total 14 marks)

18. The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram at right.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(5)

(b) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(4)

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

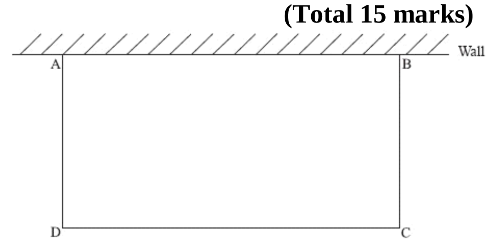
(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

(6)

(Total 15 marks)

19. The diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



Find the width of the rectangle that gives its maximum area.

(Total 6 marks)

20. A particle moves with a velocity v m s^{-1} given by $v = 25 - 4t^2$ where $t \geq 0$.

(a) The displacement, s metres, is 10 when t is 3. Find an expression for s in terms of t.

(6)

(b) Find t when s reaches its maximum value.

(3)

(c) The particle has a positive displacement for $m \leq t \leq n$. Find the value of m and the value of n.

(3)

(Total 12 marks)

21. If $f'(x) = \cos x$, and $f\left(\frac{\pi}{2}\right) = -2$, find $f(x)$.

Working:

Answer:

....

(Total 4 marks)

22. (a) Sketch the graph of $y = \pi \sin x - x$, $-3 \le x \le 3$, on millimetre square paper, using a scale of 2 cm per unit on each axis.
Label and number both axes and indicate clearly the approximate positions of the x-intercepts and the local maximum and minimum points.

(5)

(b) Find the solution of the equation $\pi \sin x - x = 0$, $x > 0$.

(1)

(c) Find the indefinite integral $\int (\pi \sin x - x)\,dx$

and hence, or otherwise, calculate the area of the region enclosed by the graph, the x-axis and the line $x = 1$.

(4)
(Total 10 marks)

23. A curve with equation $y = f(x)$ passes through the point (1, 1). Its gradient function is $f'(x) = -2x + 3$. Find the equation of the curve.

Answer:
....

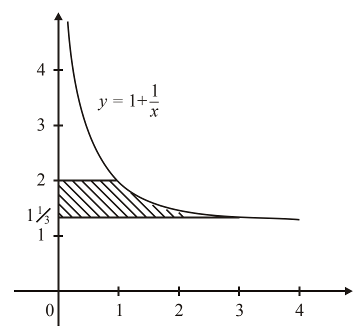
(Total 4 marks)

24. Given that $f(x) = (2x + 5)^3$ find
(a) $f'(x)$;
(b) $\int f(x)\,dx$.

Answers:
(a) ....
(b) ....

(Total 4 marks)

25. The diagram shows the graph of the function $y = 1 + \frac{1}{x}$, $0 < x \le 4$. Find the **exact** value of the area of the shaded region.



Answer:
....
....

(Total 4 marks)

26. **In this question you should note that radians are used throughout.**

(a) (i) Sketch the graph of $y = x^2 \cos x$, for $0 \le x \le 2$ making clear the approximate positions of the positive x-intercept, the maximum point and the end-points.
(ii) Write down the **approximate** coordinates of the positive x-intercept, the maximum point and the end-points.

(7)

(b) Find the **exact value** of the positive x-intercept for $0 \le x \le 2$.

(2)

Let R be the region in the first quadrant enclosed by the graph and the x-axis.
(c) (i) Shade R on your diagram.
(ii) Write down an integral which represents the area of R.

(3)

(d) Evaluate the integral in part (c)(ii), either by using a graphic display calculator, or by using the following information.

$$\frac{d}{dx}(x^2 \sin x + 2x \cos x - 2 \sin x) = x^2 \cos x.$$

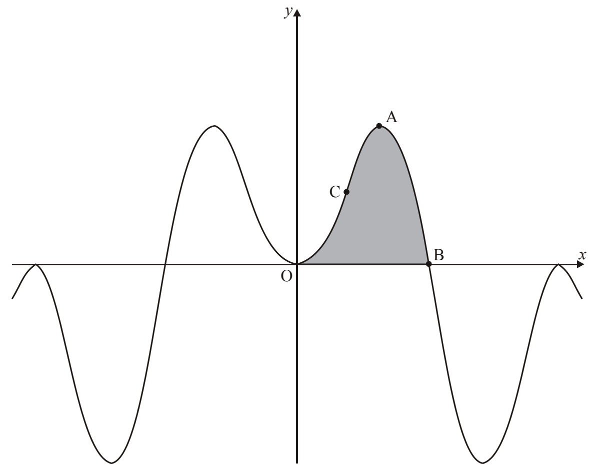
(3)
(Total 15 marks)

27. **In this part of the question, radians are used throughout.**

The function f is given by

$$f(x) = (\sin x)^2 \cos x.$$

The diagram shows part of the graph of $y = f(x)$.



The point A is a maximum point, the point B lies on the x-axis, and the point C is a point of inflexion.

(a) Give the period of f.

(1)

(b) From consideration of the graph of $y = f(x)$, find **to an accuracy of one significant figure** the range of f.

(1)

(c) (i) Find $f'(x)$.

(ii) Hence show that at the point A, $\cos x = \sqrt{\frac{1}{3}}$.

(iii) Find the exact maximum value.

(9)

(d) Find the exact value of the x-coordinate at the point B.

(1)

(e) (i) Find $\int f(x)\,dx$.

(ii) Find the area of the shaded region in the diagram.

(4)

(f) Given that $f''(x) = 9(\cos x)^3 - 7\cos x$, find the x-coordinate at the point C.

(4)

(Total 20 marks)

28. Let $f'(x) = 1 - x^2$. Given that $f(3) = 0$, find $f(x)$.

Working:

Answer:

....

(Total 4 marks)

29. **Note**: Radians are used throughout this question.

(a) Draw the graph of $y = \pi + x\cos x$, $0 \le x \le 5$, on millimetre square graph paper, using a scale of 2 cm per unit. Make clear

(i) the integer values of x and y on each axis;

(ii) the approximate positions of the x-intercepts and the turning points.

(5)

(b) **Without the use of a calculator**, show that π is a solution of the equation $\pi + x\cos x = 0$.

(3)

(c) Find another solution of the equation $\pi + x\cos x = 0$ for $0 \le x \le 5$, giving your answer to **six** significant figures.

(2)

(d) Let R be the region enclosed by the graph and the axes for $0 \le x \le \pi$. Shade R on your diagram, and write down an integral which represents the area of R.

(2)

(e) Evaluate the integral in part (d) to an accuracy of **six** significant figures. (If you consider it necessary, you can make use of the result $\frac{d}{dx}(x\sin x + \cos x) = x\cos x$.)

(3)

(Total 15 marks)

30. Find

(a) $\int \sin(3x+7)\mathrm{d}x$;

(b) $\int e^{-4x}\mathrm{d}x$.

Answers:
(a) ....
(b) ....

(Total 4 marks)

31. The derivative of the function f is given by $f'(x) = \frac{1}{1+x} - 0.5\sin x$, for $x \neq -1$.

The graph of f passes through the point (0, 2). Find an expression for $f(x)$.

Answer:
....

(Total 6 marks)

32. Consider functions of the form $y = \mathrm{e}^{-kx}$

(a) Show that $\int_0^1 \mathrm{e}^{-kx}\mathrm{d}x = \frac{1}{k}(1 - \mathrm{e}^{-k})$.

(3)

(b) Let $k = 0.5$

(i) Sketch the graph of $y = \mathrm{e}^{-0.5x}$, for $-1 \leq x \leq 3$, indicating the coordinates of the y-intercept.

(ii) Shade the region enclosed by this graph, the x-axis, y-axis and the line $x = 1$.

(iii) Find the area of this region.

(5)

(c) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of k, where $y = \mathrm{e}^{-kx}$.

The point P(1, 0.8) lies on the graph of the function $y = \mathrm{e}^{-kx}$.

(ii) Find the value of k in this case.

(iii) Find the gradient of the tangent to the curve at P.

(5)

(Total 13 marks)

33. Let $f(x) = \sqrt{x^3}$. Find

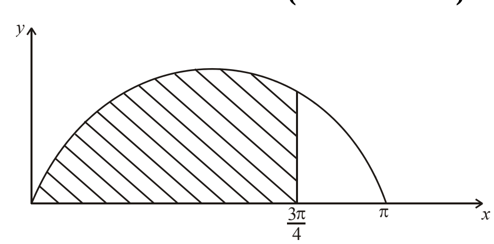
(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Answers:
(a) ....
(b) ....

(Total 6 marks)

34. The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \frac{3\pi}{4}$.

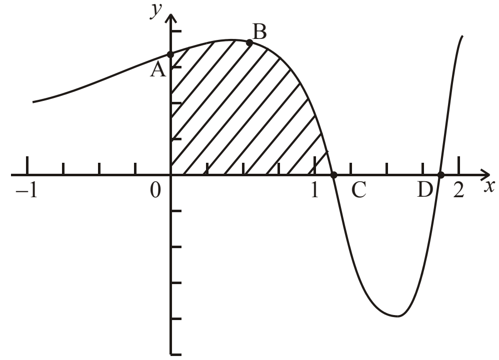


Given that $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

Answer:
....

(Total 6 marks)

35. The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A. **(2)**

(b) The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k. **(2)**

(c) (i) Write down the y-coordinate of B.

(ii) Find $\dfrac{dy}{dx}$.

(iii) Hence, show that at B, $x = \ln \dfrac{\pi}{2}$.

(6)

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

(5)

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

(3)

(Total 18 marks)

36. Given that $\int_1^3 g(x)dx = 10$, deduce the value of

(a) $\int_1^3 \frac{1}{2} g(x)dx$;

(b) $\int_1^3 (g(x)+4)dx$.

Answers:
(a) ....
(b) ....

(Total 6 marks)

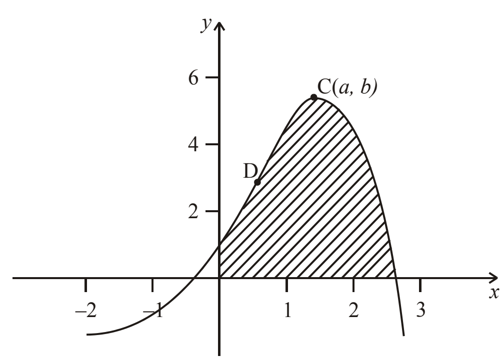
37. Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f(-\frac{\pi}{4}) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

(3)

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \le x \le 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\dfrac{dy}{dx}$.

(3)

(c) Find the **exact** value of a and of b.

(4)

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(5)

(e) Find the area of the shaded region. **(2)**

(Total 17 marks)

38. It is given that $\frac{dy}{dx} = x^3 + 2x - 1$ and that $y = 13$ when $x = 2$.

Find y in terms of x.

Answer:
....

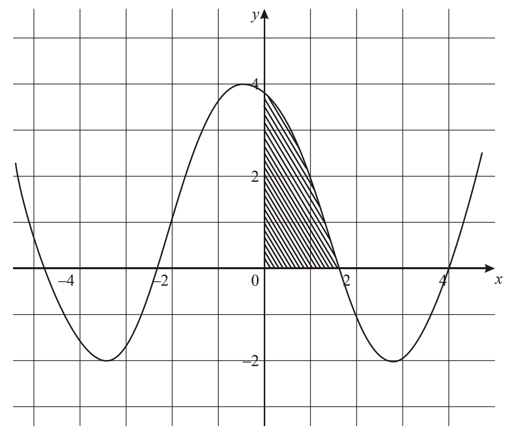
(Total 6 marks)

39. (a) Find $\int(1 + 3\sin(x + 2))dx$.

(b) The diagram shows part of the graph of the function $f(x) = 1 + 3\sin(x + 2)$.
The area of the shaded region is given by

$$\int_0^a f(x)dx.$$

Find the value of a.

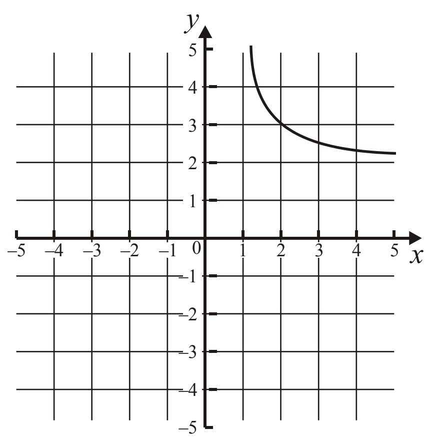


Answers:
(a) ....
(b) ....

(Total 6 marks)

40. (a) Consider the function $f(x) = 2 + \frac{1}{x-1}$. The diagram below is a sketch of part of the graph of $y = f(x)$.

Copy and complete the sketch of $f(x)$.



(2)

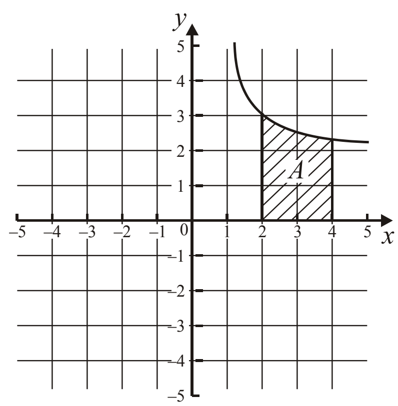
(b) (i) Write down the x-intercepts and y-intercepts of $f(x)$.
(ii) Write down the equations of the asymptotes of $f(x)$.

(4)

(c) (i) Find $f'(x)$.
(ii) There are no maximum or minimum points on the graph of $f(x)$.
Use your expression for $f'(x)$ to explain why.

(3)

The region enclosed by the graph of $f(x)$, the x-axis and the lines $x = 2$ and $x = 4$, is labelled A, as shown at right.



(d) (i) Find $\int f(x)\,dx$.
(ii) Write down an expression that represents the area labelled A.
(iii) Find the area of A.

(7)
(Total 16 marks)

41. The derivative of the function f is given by $f'(x) = e^{-2x} + \frac{1}{1-x}$, $x < 1$.

The graph of $y = f(x)$ passes through the point $(0, 4)$. Find an expression for $f(x)$.

Working:

Answer:
…………………………………………….......

(Total 6 marks)

42. Let f be a function such that $\int_0^3 f(x)\,dx = 8$.

(a) Deduce the value of

(i) $\int_0^3 2f(x)\,dx$;

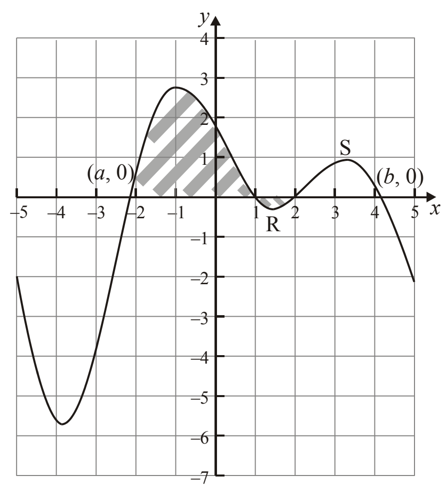
(ii) $\int_0^3 (f(x)+2)\,dx$.

(b) If $\int_c^d f(x-2)\,dx = 8$, write down the value of c and of d.

Answers:
(a) (i) ....
(ii) ....
(b) c = ..., d = ...

(Total 6 marks)

43. Let $h(x) = (x-2)\sin(x-1)$ for $-5 \le x \le 5$. The curve of $h(x)$ is shown at right. There is a minimum point at R and a maximum point at S. The curve intersects the x-axis at the points $(a, 0)$ $(1, 0)$ $(2, 0)$ and $(b, 0)$.



(a) Find the exact value of

(i) a;

(ii) b.

(2)

The regions between the curve and the x-axis are shaded for $a \le x \le 2$ as shown.

(b) (i) Write down an expression which represents the **total** area of the shaded regions.

(ii) Calculate this total area.

(5)

(c) (i) The y-coordinate of R is –0.240. Find the y-coordinate of S.

(ii) Hence or otherwise, find the range of values of k for which the equation $(x-2)\sin(x-1) = k$ has **four** distinct solutions.

(4)

(Total 11 marks)

44. Let $f(x) = \dfrac{1}{1+x^2}$.

(a) Write down the equation of the horizontal asymptote of the graph of f.

(1)

(b) Find $f'(x)$.

(3)

(c) The second derivative is given by $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$.

Let A be the point on the curve of f where the gradient of the tangent is a maximum. Find the x-coordinate of A.

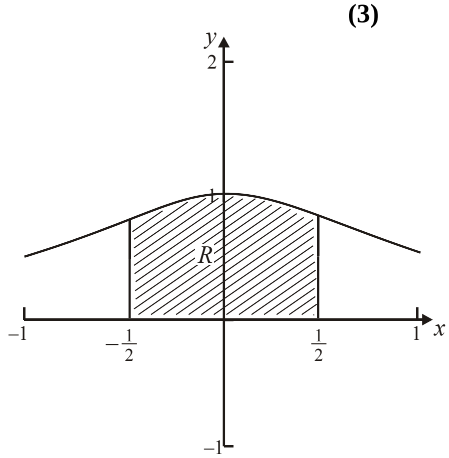
(4)

(d) Let R be the region under the graph of f, between $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$,

as shaded in the diagram at right

Write down the definite integral which represents the area of R.

(2)



(Total 10 marks)

45. The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f''(x)$.

(b) Write down $\int f(x)\mathrm{d}x$.

....

....

....

....

....

(Total 6 marks)

46. Let $f(x) = (3x + 4)^5$. Find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Answers:
(a) ....
(b) ....

(Total 6 marks)

47. The curve $y = f(x)$ passes through the point (2, 6).

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 5$, find y in terms of x.

Answer:
....

(Total 6 marks)

48. The table below shows some values of two functions, f and g, and of their derivatives f' and g'.

x	1	2	3	4
$f(x)$	5	4	–1	3
$g(x)$	1	–2	2	–5
$f'(x)$	5	6	0	7
$g'(x)$	–6	–4	–3	4

Calculate the following.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}(f(x) + g(x))$, when $x = 4$;

(b) $\int_1^3 (g'(x) + 6)\mathrm{d}x$.

Answers:
(a) ....
(b) ....

(Total 6 marks)

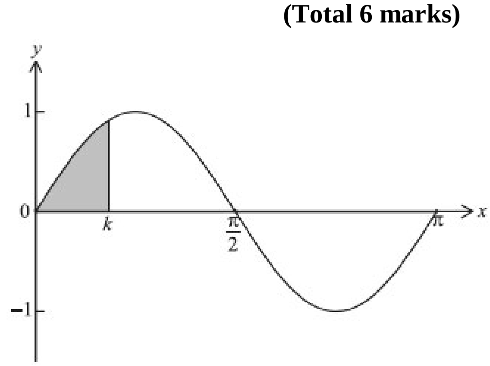
49. Given $\int_3^k \frac{1}{x-2}\mathrm{d}x = \ln 7$, find the value of k.

(Total 6 marks)

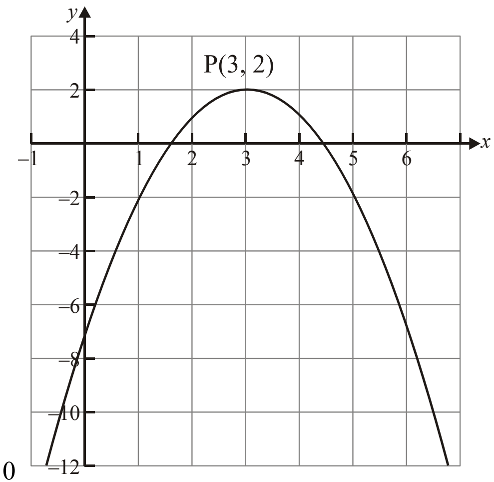
50. The graph of $y = \sin 2x$ from $0 \le x \le \pi$ is shown at right.

The area of the shaded region is 0.85. Find the value of k.

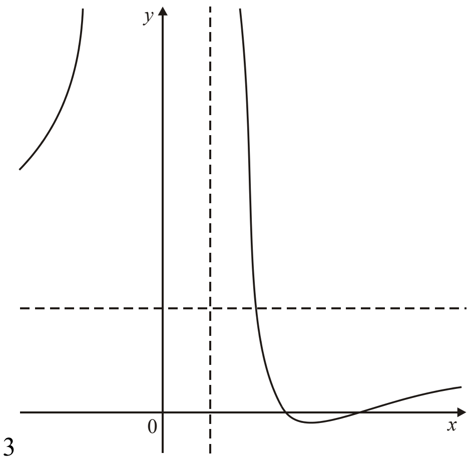
(Total 6 marks)



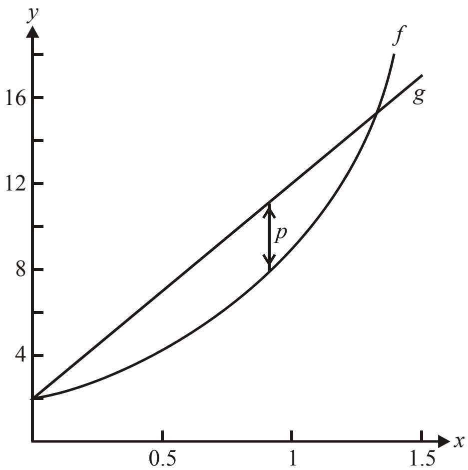
Diagrams for Calc Practice problems (they don't print when text wrapping is on).



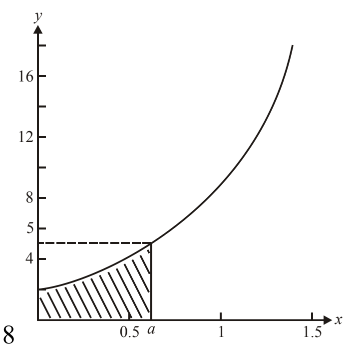
10



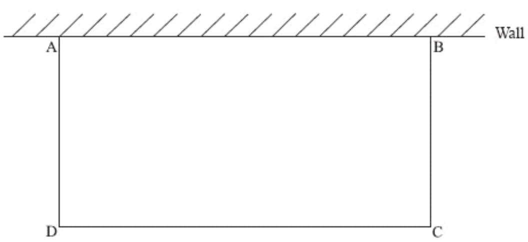
13



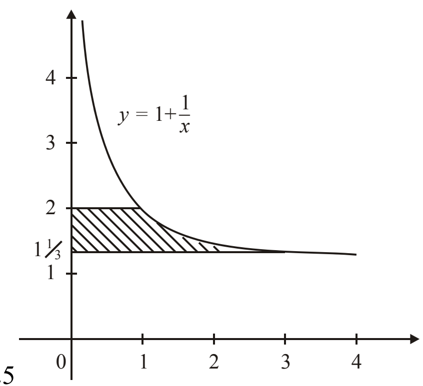
17



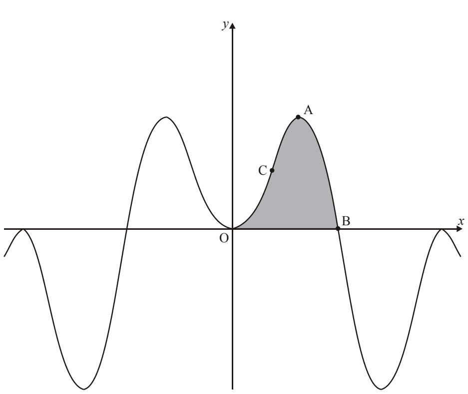
18



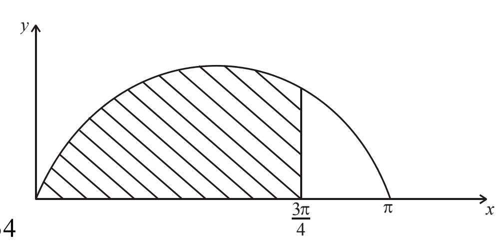
19



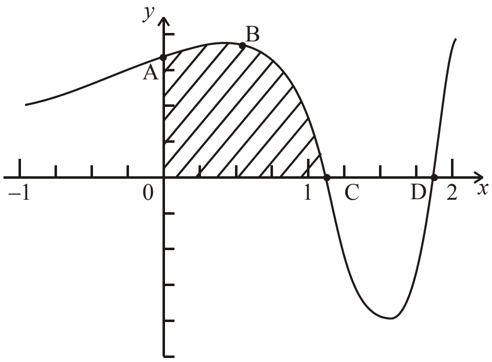
25



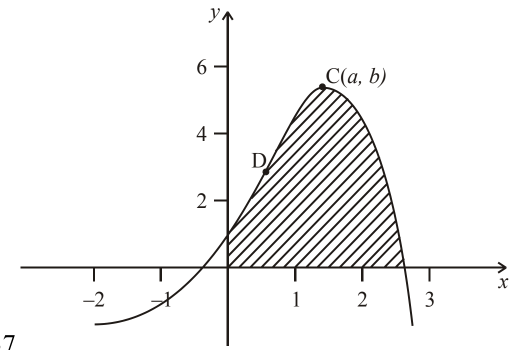
27



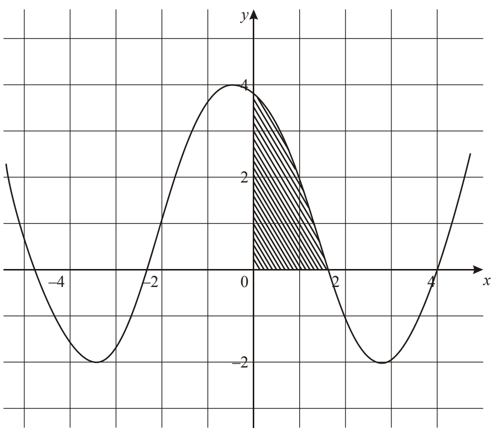
34



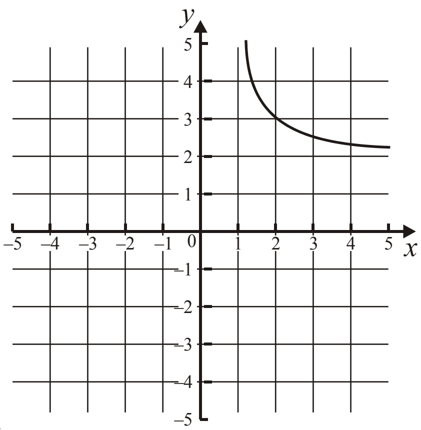
35



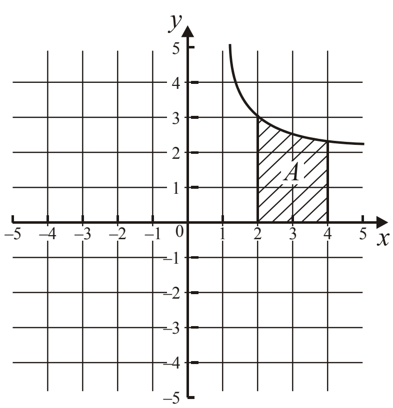
37



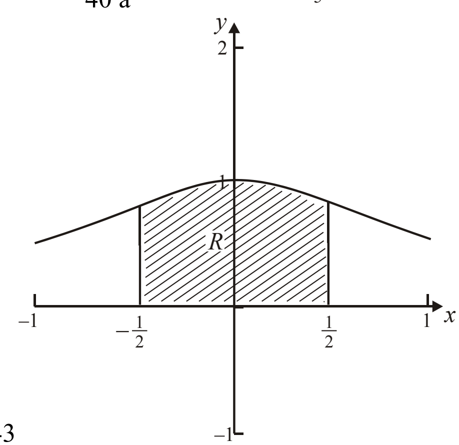
39



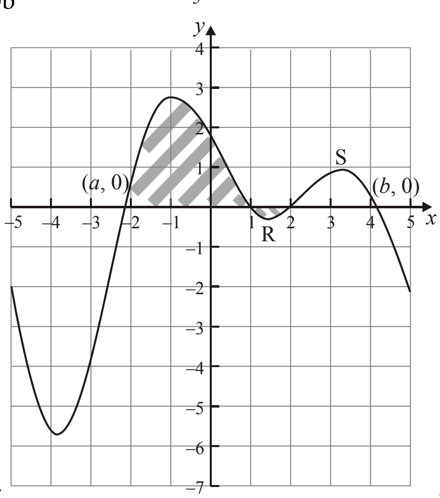
40 a



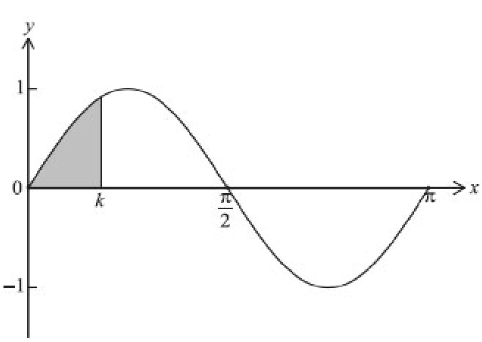
40b



43



44



50

SL Calculus Practice Problems - MarkScheme

1. $y = \sin(2x - 1)$

$\frac{dy}{dx} = 2\cos(2x - 1)$ (A1)(A1)

At $\left(\frac{1}{2}, 0\right)$, the gradient of the tangent $= 2\cos 0$ (A1)

$= 2$ (A1) (C4)

2. (a) (i) $a = -3$ (A1)

(ii) $b = 5$ (A1) 2

(b) (i) $f'(x) = -3x^2 + 4x + 15$ (A2)

(ii) $-3x^2 + 4x + 15 = 0$

$-(3x + 5)(x - 3) = 0$ (M1)

$x = -\frac{5}{3}$ or $x = 3$ (A1)(A1)

OR

$x = -\frac{5}{3}$ or $x = 3$ (G3)

(iii) $x = 3 \Rightarrow f(3) = -3^3 + 2(3^2) + 15(3)$ (M1)

$= -27 + 18 + 45 = 36$ (A1)

OR

$f(3) = 36$ (G2) 7

(c) (i) $f'(x) = 15$ at $x = 0$ (M1)

Line through (0, 0) of gradient 15

$\Rightarrow y = 15x$ (A1)

OR

$y = 15x$ (G2)

(ii) $-x^3 + 2x^2 + 15x = 15x$ (M1)

$\Rightarrow -x^3 + 2x^2 = 0$

$\Rightarrow -x^2(x - 2) = 0$

$\Rightarrow x = 2$ (A1)

OR

$x = 2$ (G2) 4

(d) Area =115 (3 sf) (G2)

OR

Area $= \int_0^6 (-x^3 + 2x^2 + 15x)\,dx = \left[-\frac{x^4}{4} + 2\frac{x^3}{3} + 15\frac{x^2}{2}\right]_0^5$ (M1)

$= \frac{1375}{12} = 115$ (3 sf) (A1) 2

[15]

3. (a) (i) $v(0) = 50 - 50e^0 = 0$ (A1)

(ii) $v(10) = 50 - 50e^{-2} = 43.2$ (A1) 2

(b) (i) $a = \frac{dv}{dt} = -50(-0.2e^{-0.2t})$ (M1)

$= 10e^{-0.2t}$ (A1)

(ii) $a(0) = 10e^0 = 10$ (A1) 3

(c) (i) $t \to \infty \Rightarrow v \to 50$ (A1)

(ii) $t \to \infty \Rightarrow a \to 0$ (A1)

(iii) when $a = 0$, v is constant at 50 (R1) 3

(d) (i) $y = \int v\,dt$ (M1)

$= 50t - \dfrac{e^{-0.2t}}{-0.2} + k$ (A1)

$= 50t + 250e^{-0.2t} + k$ (AG)

(ii) $0 = 50(0) + 250e^{0} + k = 250 + k$ (M1)

$\Rightarrow k = -250$ (A1)

(iii) Solve $250 = 50t + 250e^{-0.2t} - 250$ (M1)

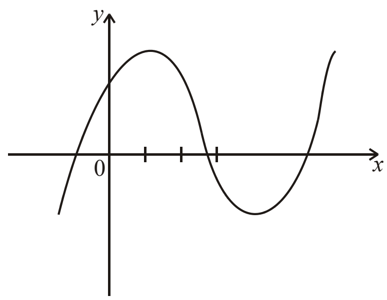
$\Rightarrow 50t + 250e^{-0.2t} - 500 = 0$

$\Rightarrow t + 5e^{-0.2t} - 10 = 0$

$\Rightarrow t = 9.207\ s$ (G2) 7

[15]

4. METHOD 1



Using gdc coordinates of maximum are (0.667, 26.9) (G3)(G3)(C6)

METHOD 2

At maximum $\dfrac{dy}{dx} = 3x^2 - 20x + 12 = 0 = (3x - 2)(x - 6)$ (M1)(A1)(M1)

=> $x = \dfrac{2}{3}$ must be where maximum occurs (A1)

$x = \dfrac{2}{3}$ => $y = \left(\dfrac{2}{3}\right)^3 - 10\left(\dfrac{2}{3}\right)^2 + 12\left(\dfrac{2}{3}\right) + 23 = \dfrac{725}{27}$ (= 26.9, 3 sf) (M1)(A1)

Maximum at $\left(\dfrac{2}{3}, \dfrac{725}{27}\right)$ (C4)(C2)

[6]

5. (a) $\dfrac{ds}{dt} = 30 - at$ => $s = 30t - a\dfrac{t^2}{2} + C$ (A1)(A1)(A1)

Note: *Award (A1) for 30t, (A1) for* $a\dfrac{t^2}{2}$*, (A1) for C.*

$t = 0$ => $s = 30(0) - a\dfrac{(0^2)}{2} + C = 0 + C$ => $C = 0$ (M1)

=> $s = 30t - \dfrac{1}{2}at^2$ (A1) 5

(b) (i) vel = 30 – 5(0) = 30 m s^{-1} (A1)

(ii) Train will stop when $0 = 30 - 5t$ => $t = 6$ (M1)

Distance travelled $= 30t - \dfrac{1}{2}at^2$

$= 30(6) - \dfrac{1}{2}(5)(6^2)$ (M1)

= 90m (A1)

90 < 200 => train stops before station. (R1)(AG) 5

(c) (i) $0 = 30 - at \Rightarrow t = \frac{30}{a}$ (A1)

(ii) $30\left(\frac{30}{a}\right) - \frac{1}{2}(a)\left(\frac{30}{a}\right)^2 = 200$ (M1)(M1)

Note: *Award (M1) for substituting* $\frac{30}{a}$, *(Ml) for setting equal to 200.*

$\Rightarrow \frac{900}{a} - \frac{450}{a} = \frac{450}{a} = 200$ (A1)

$\Rightarrow a = \frac{450}{200} = \frac{9}{4} = 2.25 \text{ m s}^{-2}$ (A1) 5

Note: *Do not penalize lack of units in answers.*

[15]

6. (i) At $x = a$, $h(x) = a^{\frac{1}{5}}$

$h'(x) = \frac{1}{5}x^{-\frac{4}{5}} \Rightarrow h'(a) = \frac{1}{5a^{\frac{4}{5}}}$ = gradient of tangent (A1)

$\Rightarrow y - a^{\frac{1}{5}} = \frac{1}{5a^{\frac{4}{5}}}(x - a) = \frac{1}{5a^{\frac{4}{5}}}x - \frac{1}{5}a^{\frac{1}{5}}$ (M1)

$\Rightarrow y = \frac{1}{5a^{\frac{4}{5}}}x + \frac{4}{5}a^{\frac{1}{5}}$ (A1)

(ii) tangent intersects x-axis $\Rightarrow y = 0$

$\Rightarrow \frac{1}{5a^{\frac{4}{5}}}x = -\frac{4}{5}a^{\frac{1}{5}}$ (M1)

$\Rightarrow x = 5a^{\frac{4}{5}}\left(-\frac{4}{5}a^{\frac{1}{5}}\right) = -4a$ (M1)(AG) 5

[5]

7. (a) (i) When $t = 0$, $v = 50 + 50e^{0}$ (A1)

$= 100 \text{ m s}^{-1}$ (A1)

(ii) When $t = 4$, $v = 50 + 50e^{-2}$ (A1)

$= 56.8 \text{ m s}^{-1}$ (A1) 4

(b) $v = \frac{ds}{dt} \Rightarrow s = \int v\,dt$

$\int_0^4 \left(50 + 50e^{-0.5t}\right)dt$ (A1)(A1)(A1) 3

Note: *Award (A1) for each limit in the correct position and (A1) for the function.*

(c) Distance travelled in 4 seconds $= \int_0^4 \left(50 + 50e^{-0.5t}\right)dt$

$= [50t - 100e^{-0.5t}]_0^4$ (A1)

$= (200 - 100e^{-2}) - (0 - 100e^{0})$

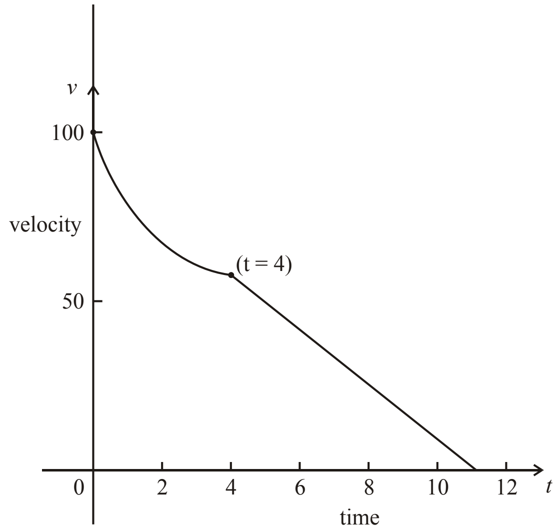
$= 286$ m (3 sf) (A1)

Note: *Award first (A1) for* $[50t - 100e^{-0.5t}]$*, ie limits not required.*

OR

Distance travelled in 4 seconds = 286 m (3 sf) (G2) 2

(d)



Notes: *Award (A1) for the exponential part, (A1) for the straight line through (11, 0), Award (A1) for indication of time on x-axis* **and** *velocity on y-axis,*
(A1) for scale on x-axis **and** *y-axis.*
Award (A1) for marking the point where t = 4.

5

(e) Constant rate = $\frac{56.8}{7}$ (M1)

$= 8.11 \text{ m s}^{-2}$ (A1) 2

Note: *Award (M1)(A0) for –8.11.*

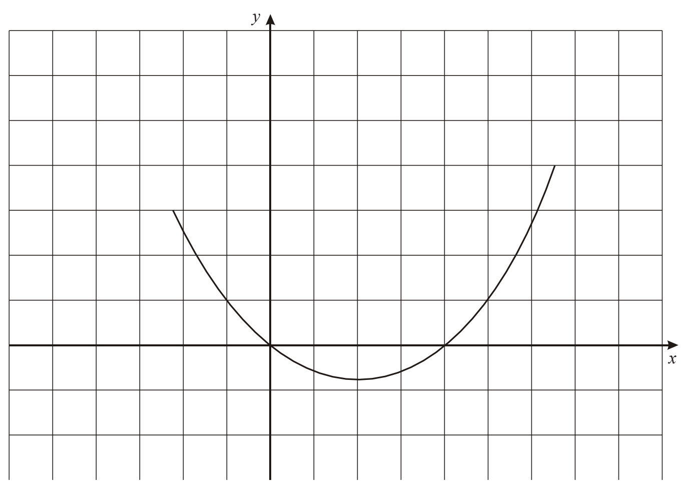
(f) distance = $\frac{1}{2}(7)(56.8)$ (M1)

= 199 m (A1) 2

Note: *Do not award* ***ft*** *in parts (e) and (f) if candidate has not used a straight line for t = 4 to t = 11 or if they continue the exponential beyond t = 4.*

[18]

8.



(A2)(A1)(A1)(A2) (C6)

Note: *Award A2 for correct shape (approximately parabolic), A1 A1 for intercepts at 0 and 4, A2 for minimum between x = 1.5 and x = 2.5.*

[6]

9. (a) (i) $f'(x) = -2e^{-2x}$ (A1)

(ii) $f'(x)$ is always negative (R1) 2

(b) (i) $y = 1 + e^{-2\times -\frac{1}{2}}$ $(= 1 + e)$ (A1)

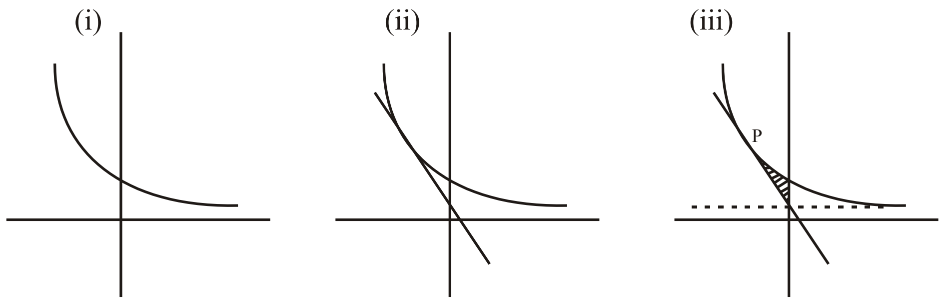
(ii) $f'\left(-\frac{1}{2}\right) = -2e^{-2\times -\frac{1}{2}}$ $(= -2e)$ (A1) 2

***Note:** In part (b) the answers do not need to be simplified.*

(c) $y - (1 + e) = -2e\left(x + \frac{1}{2}\right)$ (M1)

$y = -2ex + 1$ ($y = -5.44x + 1$) (A1)(A1) 3

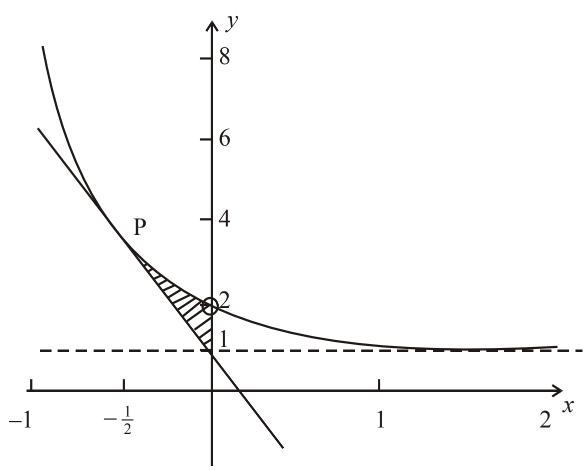
(d)



(A1)(A1)(A1)

***Notes:** Award (A1) for each correct answer. Do **not** allow **(ft)** on an incorrect answer to part (i). The correct final diagram is shown below. Do not penalize if the horizontal asymptote is missing. Axes do not need to be labelled.*

(i)(ii)(iii)



(iv) Area = $\int_{-\frac{1}{2}}^{0}[(1 + e^{-2x}) - (-2ex + 1)]dx$ (or equivalent) (M1)(M1)

***Notes:** Award (M1) for the limits, (M1) for the function.*
Accept difference of integrals as well as integral of difference. Area below line may be calculated geometrically.

Area = $\int_{-\frac{1}{2}}^{0}[(e^{-2x} + 2ex)dx$

$= \left[-\frac{1}{2}e^{-2x} + ex^2\right]_{-\frac{1}{2}}^{0}$ (A1)

$= 0.1795 \ldots = 0.180$ (3 sf) (A1)

OR

Area = 0.180 (G2) 7

[14]

10. (a) $x = 1$ (A1)1

(b) (i) $f(-1000) = 2.01$ (A1)

(ii) $y = 2$ (A1) 2

(c) $f'(x) = \dfrac{(x-1)^2(4x-13) - 2(x-1)(2x^2 - 13x + 20)}{(x-1)^4}$ (A1)(A1)

$= \dfrac{(4x^2 - 17x + 13) - (4x^2 - 26x + 40)}{(x-1)^3}$ (A1)

$= \dfrac{9x - 27}{(x-1)^3}$ (AG) 3

Notes: *Award (M1) for the **correct** use of the quotient rule, the first (A1) for the placement of the correct expressions into the quotient rule.*
Award the second (A1) for doing sufficient simplification to make the given answer reasonably obvious.

(d) $f'(3) = 0$ $\Rightarrow$ stationary (or turning) point (R1)

$f''(3) = \dfrac{18}{16} > 0$ $\Rightarrow$ minimum (R1) 2

(e) Point of inflexion $\Rightarrow f''(x) = 0 \Rightarrow x = 4$ (A1)

$x = 4 \Rightarrow y = 0$ $\Rightarrow$ Point of inflexion = (4, 0) (A1)

OR

Point of inflexion = (4, 0) (G2) 2

[10]

11. (a) $d = \int_0^4 (4t + 5 - 5e^{-t})\,dt$ (M1)(A1)(A1)(C3)

Note: *Award (M1) for* $\int$, *(A1) for **both** limits, (A1) for* $4t + 5 - 5e^{-t}$

(b) $d = [2t^2 + 5t + 5e^{-t}]_0^4$ (A1)(A1)

Note: *Award (A1) for* $2t^2 + 5t$, *(A1) for* $5e^{-t}$.

$= (32 + 20 + 5e^{-4}) - (5)$

$= 47 + 5e^{-4}$ (47.1, 3sf) (A1) (C3)

[6]

12. (a) Velocity is $\dfrac{ds}{dt}$. (M1)

$\dfrac{ds}{dt} = 10 - t$ (A1)

10 (m s^{-1}) (A1) (C3)

(b) The velocity is zero when $\dfrac{ds}{dt} = 0$ (M1)

$10 - t = 0$

$t = 10$ (secs) (A1)(C2)

(c) $s = 50$ (metres) (A1) (C1)

Note: *Do not penalize absence of units.*

[6]

13. (a) $h = 3$ (A1)

$k = 2$ (A1) 2

(b) $f(x) = -(x-3)^2 + 2$

$= -x^2 + 6x - 9 + 2$ (must be a correct expression) (A1)

$= -x^2 + 6x - 7$ (AG)1

(c) $f'(x) = -2x + 6$ (A2) 2

(d) (i) tangent gradient $= -2$ (A1)

gradient of $L = \frac{1}{2}$ (A1) (N2) 2

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ (M1)

$c = -1$. (A1)

$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$ (A2) (N2) 2

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (M1)

$2x^2 - 11x + 12 = 0$ (may be implied) (A1)

$(2x - 3)(x - 4) = 0$ (may be implied) (A1)

$x = 1.5$ (A1) (N3) 4

OR

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) (M1)

$x = 1.5$ (A3) (N3) 8

[13]

14. (a) $x = 4$ (A1)

g'' changes sign at $x = 4$ or concavity changes (R1) 2

(b) $x = 2$ (A1)

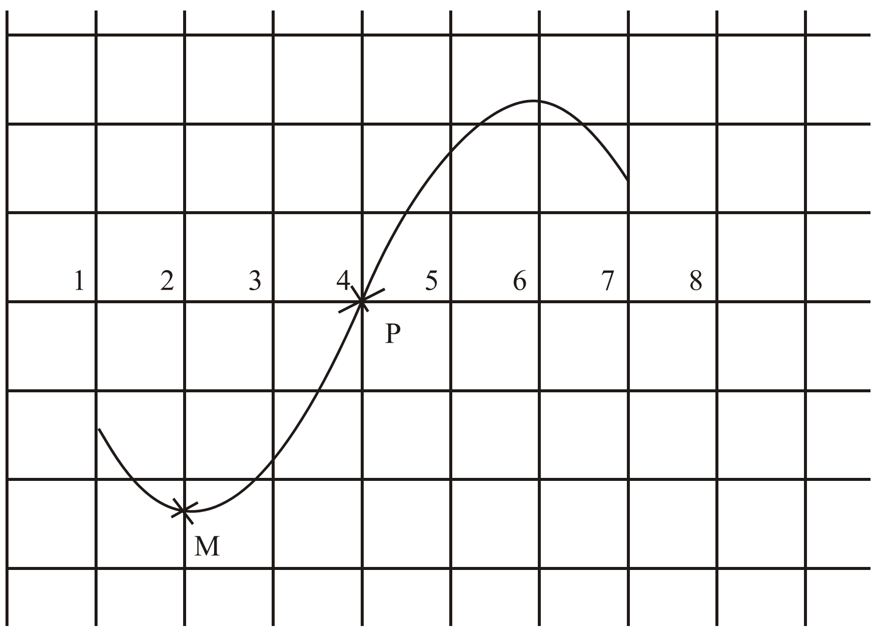
EITHER

g' goes from negative to positive (R1)

OR

$g'(2) = 0$ **and** $g''(2)$ is positive (R1) 2

(c)



(A2)(A1)(A1) 4

Note: *Award (A2) for a suitable cubic curve through (4, 0), (A1) for M at x = 2, (A1) for P at (4, 0).*

[8]

15. (a) $a = \frac{dv}{dt}$ (M1)

$= -10$ A1 3

(b) $s = \int v dt$ (M1)

$= 50t - 5t^2 + c$ A1

$40 = 50(0) - 5(0) + c \Rightarrow c = 40$ A1

$s = 50t - 5t^2 + 40$ A1 3

***Note**: Award (M1) and the first (A1) in part (b) if c is missing, but do not award the final 2 marks.*

[6]

16. (a) (i) $f'(x) = -x + 2$ A1

(ii) $f'(0) = 2$ A12

(b) Gradient of tangent at y-intercept $= f'(0) = 2$

$\Rightarrow$ gradient of normal $= \frac{1}{2}$ $(= -0.5)$ A1

Finding y-intercept is 2.5 A1

Therefore, equation of the normal is

$y - 2.5 = \sim(x - 0)$ $(y - 2.5 = -0.5x)$ M1

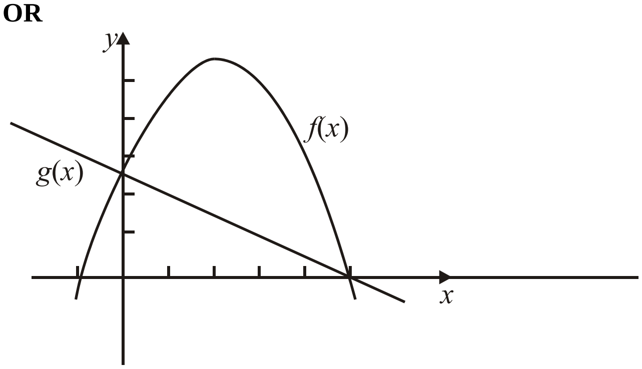
$(y = -0.5x + 2.5$ (AG) 3

(c) (i) **EITHER**

solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ (M1)A1

$\Rightarrow x = 0$ or $x = 5$ A1 2

OR



M1

Curves intersect at $x = 0, x = 5$ (A1)

So solutions to $f(x) = g(x)$ are $x = 0, x = 5$ A12

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ (A1)

$\Rightarrow -0.5x(x - 5) = 0$ M1

$\Rightarrow x = 0$ or $x = 5$ A12

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ (M2)

Other point is when $x = 5$

$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point (5, 0)) A1 2

(d) (i) Area $= \int_0^5 (f(x) - g(x))dx \left(\text{or } \int_0^5 (-0.5x^2 + 2x + 2.5)dx - \frac{1}{2} \times 5 \times 2.5 \right)$

A1A1A1 3

***Note**: Award (A1) for the integral, (A1) for both correct limits on the integral, and (A1) for the difference.*

(ii) Area = Area under curve – area under line $(A = A_1 - A_2)$ (M1)

(A1) $= \frac{50}{3}, A_2 = \frac{25}{4}$

Area $= \frac{50}{3} - \frac{25}{4} = \frac{125}{12}$ (or 10.4 (3sf) A12

[16]

17. (a) (i) $p = (10x + 2) - (1 + e^{2x})$ A22

Note: Award (A1) for $(l + e^{2x}) - (10x + 2)$.

(ii) $\frac{dp}{dx} = 10 - 2e^{2x}$ A1A1

$\frac{dp}{dx} = 0 \ (10 - 2e^{2x} = 0)$ M1

$x = \frac{\ln 5}{2}$ (= 0.805) A14

(b) (i) **METHOD 1**

$x = 1 + e^{2x}$ M1

$\ln(x - 1) = 2y$ A1

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right)$ A13

METHOD 2

$y - 1 = e^{2x}$ A1

$\frac{\ln(y-1)}{2} = x$ M1

$f^{-1}(x) = \frac{\ln(x-1)}{2} \left(\text{Allow } y = \frac{\ln(x-1)}{2}\right)$ A13

(ii) $a = \frac{\ln(5-1)}{2} \left(= \frac{1}{2}\ln 2^2\right)$ M1

$= \frac{1}{2} \times 2\ln 2$ A1

$= \ln 2$ AG2

(c) Using $V = \int_a^b \pi y^2 \, dx$ (M1)

Volume $= \int_0^{\ln 2} \pi(1 + e^{2x})^2 \, dx \quad \left(\text{or} \int_0^{0.805} \pi(1 + e^{2x})^2 \, dx\right)$ A2 3

[14]

18. (a) (i) $p = -2 \quad q = 4$ (or $p = 4, q = -2$) (A1)(A1)(N1)(N1)

(ii) $y = a(x+2)(x-4)$

$8 = a(6+2)(6-4)$ (M1)

$8 = 16a$

$a = \frac{1}{2}$ (A1)(N1)

(iii) $y = \frac{1}{2}(x+2)(x-4)$

$y = \frac{1}{2}(x^2 - 2x - 8)$

$y = \frac{1}{2}x^2 - x - 4$ (A1) (N1) 5

(b) (i) $\frac{dy}{dx} = x - 1$ (A1) (N1)

(ii) $x - 1 = 7$ (M1)

$x = 8, \ y = 20 \ (\text{P is } (8, 20))$ (A1)(A1) (N2) 4

(c) (i) when $x = 4$, gradient of tangent is $4 - 1 = 3$ (may be implied) (A1) gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4)$ $\left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1)(N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (-2.67) (A1) (N2) 6

[15]

19. **METHOD 1**

$l + 2w = 60$ (M1)

$l = 60 - 2w$ (A1)

$A = w(60 - 2w)$ $(= 60w - 2w^2)$ (A1)

$\frac{dA}{dw} = 60 - 4w$ (A1)

Using $\frac{dA}{dw} = 0$ $(60 - 4w = 0)$ (M1)

$w = 15$ (A1) (C6)

METHOD 2

$w + 2l = 60$ (A1)

$w = 60 - 2l$ (A1)

$A = l(60 - 2l)$ $(= 60l - 2l^2)$ (A1)

$\frac{dA}{dl} = 60 - 4l$ (A1)

Using $\frac{dA}{dl} = 0$ $(60 - 4l = 0)$ (M1)

$l = 15$

$w = 30$ (A1) (C6)

[6]

20. (a) $s = 25t - \frac{4}{3}t^3 + c$ (M1)(A1)(A1)

***Note**: Award no further marks if "c" is missing.*

Substituting $s = 10$ and $t = 3$ (M1)

$10 = 25 \times 3 - \frac{4}{3}(3)^3 + c$

$10 = 75 - 36 + c$

$c = -29$ (A1)

$s = 25t - \frac{4}{3}t^3 - 29$ (A1) (N3)

(b) **METHOD 1**

s is a maximum when $v = \frac{ds}{dt} = 0$ (may be implied) (M1)

$25 - 4t^2 = 0$ (A1)

$t^2 = \frac{25}{4}$

$t = \frac{5}{2}$ (A1) (N2)

METHOD 2

Using maximum of s ($12\frac{2}{3}$, may be implied) (M1)

$25t - \frac{4}{3}t^3 - 29 = 12\frac{2}{3}$ (A1)

t = 2.5 (A1) (N2)

(c) $25t - \frac{4}{3}t^3 - 29 > 0$ (accept equation) (M1)

$m = 1.27$, $n = 3.55$ (A1)(A1) (N3)

[12]

21. $f'(x) = \cos x \Rightarrow f(x) = \sin x + C$ (M1)

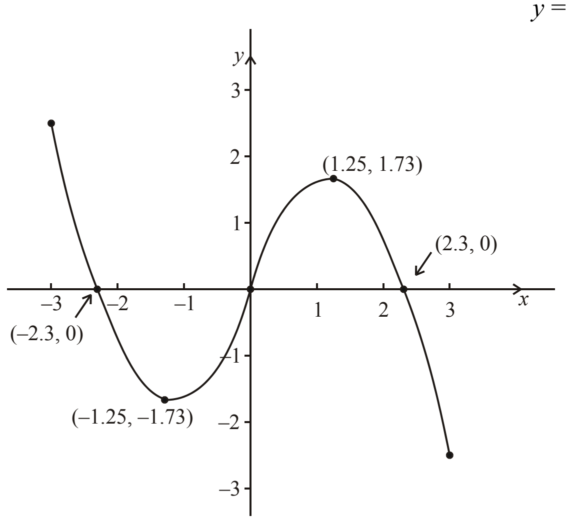
$f\left(\frac{\pi}{2}\right) = -2 \Rightarrow -2 = \sin\left(\frac{\pi}{2}\right) + C$ (M1)

$C = -3$ (A1)

$f(x) = \sin x - 3$ (A1) (C4)

[4]

22. (a) $y = \pi \sin x - x$



(A5) 5

Notes: *Award (A1) for appropriate scales marked on the axes.*
Award (A1) for the x-intercepts at (±2.3, 0).
Award (A1) for the maximum and minimum points at (±1.25, ±1.73).
Award (A1) for the end points at (±3, ±2.55).
Award (A1) for a smooth curve.
Allow some flexibility, especially in the middle three marks here.

(b) $x = 2.31$ (A1) 1

(c) $\int (\pi \sin x - x)dx = -\pi \cos x - \frac{x^2}{2} + C$ (A1)(A1)

Note: *Do not penalize for the absence of C.*

Required area = $\int_0^1 (\pi \sin x - x)dx$ (M1)

$= 0.944$ (G1)

OR area $= 0.944$ (G2) 4

[10]

23. $f'(x) = -2x + 3$

$f(x) = \frac{-2x^2}{2} + 3x + c$ (M1)

Notes: *Award (M1) for an attempt to integrate. Do not penalize the omission of c here.*

$1 = -1 + 3 + c$ (A1)

$c = -1$ (A1)

$f(x) = -x^2 + 3x - 1$ (A1) (C4)

[4]

24. (a) $f'(x) = 3(2x + 5)^2 \times 2$ (M1)(A1)

Note: *Award (M1) for an attempt to use the chain rule.*

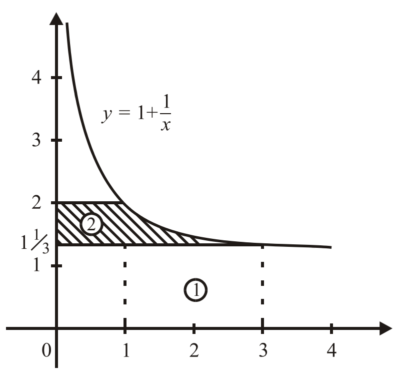
$= 6(2x + 5)^2$ (C2)

(b) $\int f(x)\mathrm{d}x = \frac{(2x+5)^4}{4 \times 2} + c$ (A2) (C2)

Note: *Award (A1) for $(2x + 5)^4$ and (A1) for /8.*

[4]

25.



Area $= \int_{1\frac{1}{3}}^{2} x\mathrm{d}y = \int_{1\frac{1}{3}}^{2} \frac{1}{(y-1)}\mathrm{d}y$ (M1)(A1)

$= \left[\ln(y-1)\right]_{1\frac{1}{3}}^{2}$

$= \ln 1 - \ln\frac{1}{3}$ (A1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1 + \frac{1}{x}\right)\mathrm{d}x = [x + \ln x]_1^3$

$= (3 + \ln 3) - (1 + \ln 1)$ (M1)

$= 2 + \ln 3$ (A1)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 2 + \ln 3 - 2\frac{2}{3} + \frac{2}{3}$ (M1)

$= \ln 3$ (A1) (C4)

OR

Area from $x = 1$ to $x = 3$, $A = \int_1^3 \left(1 + \frac{1}{x}\right)\mathrm{d}x$ (M1)

$A = 3.0986\ \ldots$ (G0)

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

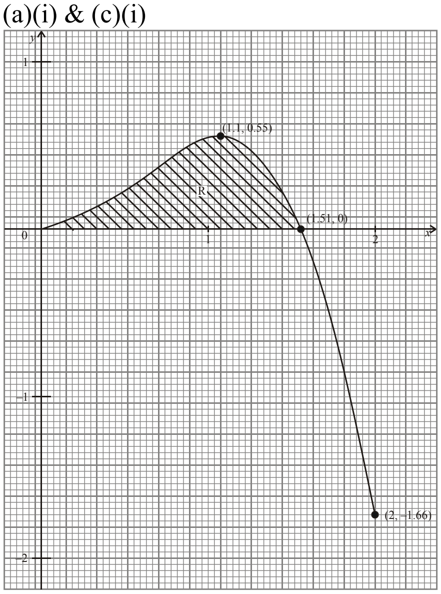
Shaded area $= 3.0986 - 2\frac{2}{3} + \frac{2}{3}$ (M1)

$= 1.10$ (3 sf) (A1) (C4)

Notes: *An exact value is required. If candidates have obtained the answer 1.10, and shown their working, award marks as above. However, if they do not show their working, award (G2) for the correct answer of 1.10.*
Award no marks for the giving of 3.10 as the final answer.

[4]

26. (a)(i) & (c)(i)



(A3)

Notes: *The sketch does* ***not*** *need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and (2, –1.66) should be indicated in some way.*
Award (A1) for the correct shape.
Award (A2) for 3 or 4 correctly indicated points, (A1) for 1 or 2 points.

(ii) Approximate positions are
positive x-intercept (1.57, 0) (A1)
maximum point (1.1, 0.55) (A1)
end points (0, 0) and (2, –1.66) (A1)(A1) 7

(b) $x^2 \cos x = 0$ $\quad x \neq 0$ ⇒ $\cos x = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (A1) 2

Note: *Award (A2) if answer correct.*

(c) (i) see graph (A1)

(ii) $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$ (A2) 3

Note: *Award (A1) for limits, (A1) for rest of integral correct (do not penalize missing dx).*

(d) Integral = 0.467 (G3)

OR

Integral $= \left[x^2 \sin x + 2x \cos x - 2 \sin x\right]_0^{\pi/2}$ (M1)

$= \left[\frac{\pi^2}{4}(1) + 2\left(\frac{\pi}{2}\right)(0) - 2(1)\right] - [0 + 0 - 0]$ (M1)

$= \frac{\pi}{2} - 2$ (exact) **or** 0.467 (3 sf) (A1) 3

[15]

27. (a) From graph, period = 2π (A1)1

(b) Range = $\{y \mid -0.4 < y < 0.4\}$ (A1) 1

(c) (i) $f'(x) = \frac{d}{dx}\{\cos x (\sin x)^2\}$

$= \cos x (2 \sin x \cos x) - \sin x (\sin x)^2$ **or** $-3 \sin^3 x + 2 \sin x$ (M1)(A1)(A1)

Note: *Award (M1) for using the product rule and (A1) for each part.*

(ii) $f'(x) = 0$ (M1)

$\Rightarrow \sin x\{2 \cos x - \sin^2 x\} = 0$ **or** $\sin x\{3 \cos x - 1\} = 0$ (A1)

$\Rightarrow 3 \cos^2 x - 1 = 0$

$\Rightarrow \cos x = \pm\sqrt{\left(\frac{1}{3}\right)}$ (A1)

At A, $f(x) > 0$, hence $\cos x = \sqrt{\left(\frac{1}{3}\right)}$ (R1)(AG)

(iii) $f(x) = \sqrt{\left(\frac{1}{3}\right)\left(1 - \left(\sqrt{\left(\frac{1}{3}\right)}\right)^2\right)}$ (M1)

$= \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{9}\sqrt{3}$ (A1) 9

(d) $x = \frac{\pi}{2}$ (A1) 1

(e) (i) $\int (\cos x)(\sin x)^2 \, dx = \frac{1}{3}\sin^3 x + c$ (M1)(A1)

(ii) Area = $\int_0^{\pi/2} (\cos x)(\sin x)^2 \, dx = \frac{1}{3}\left\{\left(\sin \frac{\pi}{2}\right)^3 - (\sin 0)^3\right\}$ (M1)

$= \frac{1}{3}$ (A1) 4

(f) At C $f''(x) = 0$ (M1)

$\Leftrightarrow 9 \cos^3 x - 7 \cos x = 0$

$\Leftrightarrow \cos x(9 \cos^2 x - 7) = 0$ (M1)

$\Rightarrow x = \frac{\pi}{2}$ (reject) ***or*** $x = \arccos \frac{\sqrt{7}}{3} = 0.491$ (3 sf) (A1)(A1) 4

[20]

28. $f'(x) = 1 - x^2$

$f(x) = \int (1 - x^2) dx = x - \frac{x^3}{3} + C$ (A1)

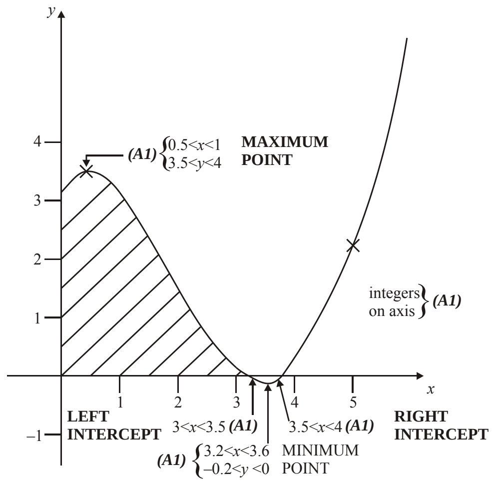
$f(3) = 0 \Rightarrow 3 - 9 + C = 0$ (M1)

$\Rightarrow c = 6$ (A1)

$f(x) = x - \frac{x^3}{3} + 6$ (A1)

[4]

29. (a)



5

(b) π is a solution if and only if $\pi + \pi \cos \pi = 0$. (M1)
Now $\pi + \pi \cos \pi = \pi + \pi(-1)$ (A1)
$= 0$ (A1) 3

(c) By using appropriate calculator functions $x = 3.696\ 722\ 9...$ (M1)
$\Rightarrow x = 3.69672$ (6sf) (A1) 2

(d) See graph: (A1)
$\int_0^\pi (\pi + x\cos x)\mathrm{d}x$ (A1) 2

(e) **EITHER** $\int_0^\pi (\pi + x\cos x)\mathrm{d}x = 7.86960$ (6 sf) (A3) 3

Note: *This answer assumes appropriate use of a calculator eg*

'fnInt': $\begin{cases} fnInt(Y_1, X, 0, \pi) = 7.869604401 \\ with\ Y_1 = \pi + x\cos x \end{cases}$

OR $\int_0^\pi (\pi + x\cos x)\mathrm{d}x = [\pi x + x\sin x + \cos x]_0^\pi$

$= \pi(\pi - 0) + (\pi \sin \pi - 0 \times \sin 0) + (\cos \pi - \cos 0)$ (A1)
$= \pi^2 + 0 + -2 = 7.86960$ (6 sf) (A1) 3

[15]

30. ***Note:*** *Do not penalize for the omission of C.*

(a) $\int \sin(3x+7)\mathrm{d}x = -\frac{1}{3}\cos(3x+7) + C$ (A1)(A1) (C2)

Note: *Award (A1) for* $\frac{1}{3}$, *(A1) for –cos (3x + 7).*

(b) $\int \mathrm{e}^{-4x}\mathrm{d}x = -\frac{1}{4}\mathrm{e}^{-4x} + C$ (A1)(A1) (C2)

Note: *Award (A1) for* $-\frac{1}{4}$, *(A1) for* e^{-4x}.

[4]

31. $f(x) = \int\left(\frac{1}{x+1} - 0.5\sin x\right)\mathrm{d}x$ (M1)
$= \ln|x+1| + 0.5\cos x + c$ (A1)(A1)(A1)
$2 = \ln 1 + 0.5 + c$ (M1)
$c = 1.5$ (A1)
$f(x) = \ln|x+1| + 0.5\cos x + 1.5$ (C6)

[6]

32. (a) $\int_0^1 e^{-kx}\,dx = \left[-\frac{1}{k}e^{-kx}\right]_0^1$ (A1)

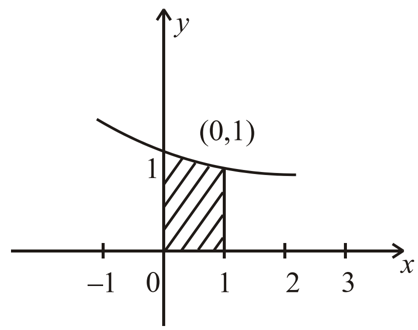
$= -\frac{1}{k}(e^{-k} - e^{0})$ (A1)

$= -\frac{1}{k}(e^{-k} - 1)$ (A1)

$= -\frac{1}{k}(1 - e^{-k})$ (AG)3

(b) $k = 0.5$

(i)



(A2)

Note: *Award (A1) for shape, and (A1) for the point (0,1).*

(ii) Shading (see graph) (A1)

(iii) Area = $\int_0^1 e^{-kx}\,dx$ for k = 0.5 (M1)

$= \frac{1}{0.5}(1 - e^{0.5})$

$= 0.787$ (3 sf) (A1)

OR

Area = 0.787 (3 sf) (G2) 5

(c) (i) $\frac{dy}{dx} = -ke^{-kx}$ (A1)

(ii) $x = 1 \quad y = 0.8 \quad \Rightarrow 0.8 = e^{-k}$ (A1)

$\ln 0.8 = -k$

$k = 0.223$ (A1)

(iii) At $x = 1$ $\frac{dy}{dx} = -0.223e^{-0.223}$ (M1)

$= -0.179$ (accept -0.178) (A1)

OR

$\frac{dy}{dx} = -0.178$ or -0.179 (G2) 5

[13]

33. $f(x) = x^{\frac{3}{2}}$ (M1)

(a) $f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}}$ (or $\frac{3}{2}\sqrt{x}$) (M1)(A1) (C3)

(b) $\int x^{\frac{3}{2}}dx = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + c$ (M1)

$= \frac{2}{5}x^{\frac{5}{2}} + c$ (or $\frac{2}{5}\sqrt{x^5} + c$) (A1)(A1) (C3)

Notes: *Do not penalize the absence of c.*

Award (A1) for $\frac{5}{2}$ *and (A1) for* $x^{\frac{5}{2}}$.

[6]

34. Area = $\int_a^b \sin x \, dx$ (M1)

$a = 0, b = \frac{3\pi}{4}$ (A1)

Area = $\int_0^{\frac{3\pi}{4}} \sin x \, dx = [-\cos x]_0^{\frac{3\pi}{4}}$ (A1)

$= \left(-\cos\frac{3\pi}{4}\right) - (-\cos 0)$ (A1)

$= -\left(-\frac{\sqrt{2}}{2}\right) - (-1)$ (A1)

$= 1 + \frac{\sqrt{2}}{2}$ (A1)(C6)

Note: *Award (G3) for a gdc answer of 1.71 or 1.707.*

[6]

35. (a) At A, $x = 0$ => $y = \sin(e^0) = \sin(1)$ (M1)

=> coordinates of A = (0,0.841) (A1)

OR

A(0, 0.841) (G2)2

(b) $\sin(e^x) = 0$ => $e^x = \pi$ (M1)

=> $x = \ln \pi$ (or $k = \pi$) (A1)

OR

$x = \ln \pi$ (or $k = \pi$) (A2) 2

(c) (i) Maximum value of sin function = 1 (A1)

(ii) $\frac{dy}{dx} = e^x \cos(e^x)$ (A1)(A1)

Note: *Award (A1) for* $\cos(e^x)$ *and (A1) for* e^x.

(iii) $\frac{dy}{dx} = 0$ at a maximum (R1)

$e^x \cos(e^x) = 0$

=> $e^x = 0$ (impossible) or $\cos(e^x) = 0$ (M1)

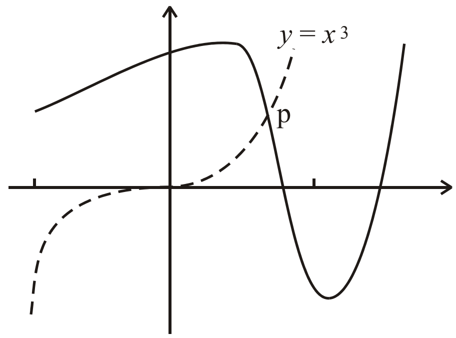
=> $e^x = \frac{\pi}{2}$ => $x = \ln\frac{\pi}{2}$ (A1)(AG) 6

(d) (i) Area = $\int_0^{\ln \pi} \sin(e^x) \, dx$ (A1)(A1)(A1)

Note: *Award (A1) for 0, (A1) for* $\ln \pi$*, (A1) for* $\sin(e^x)$.

(ii) Integral = 0.90585 = 0.906 (3 sf) (G2) 5

(e)



At P, $x = 0.87656 = 0.877$ (3 sf) (M1)(G2)3

[18]

36. (a) $\frac{1}{2} \times 10 = 5$ (M1)(A1)(C2)

(b) $\int_1^3 g(x)\,dx + \int_1^3 4\,dx$ (M1)

$\int_1^3 4\,dx [4x]_1^3$ (A1)

$= 4 \times 2 = 8$ (A1)

$\int_1^3 (g(x) + 4)\,dx = 10 + 8 = 18$ (A1) (C4)

[6]

37. (a) (i) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right) = 0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

Note: *Award (A0) for 2.36.*

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = e^x(\cos x + \sin x)$

$\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2e^x \cos x$

(c) $\frac{dy}{dx} = 0$ for a turning point $\Rightarrow 2e^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = e^{\frac{\pi}{2}}(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) = e^{\frac{\pi}{2}}$

$b = e^{\frac{\pi}{2}}$ (A1) 4

Note: *Award (M1)(A1)(A0)(A0) for a = 1.57, b = 4.81.*

(d) At D, $\frac{d^2y}{dx^2} = 0$ (M1)

$2e^x \cos x - 2e^x \sin x = 0$ (A1)

$2e^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos\frac{\pi}{4} + \sin\frac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area $= \int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

$= 7.46$ sq units (G1)

OR

Area = 7.46 sq units (G2) 2

***Note**: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.*

[17]

38. $y = \int \frac{dy}{dx} dx$ (M1)

$= \frac{x^4}{4} + \frac{2x^2}{2} - x + c$ (A1)(A1)

***Note**: Award (A1) for first 3 terms, (A1) for "+ c".*

$13 = \frac{16}{4} + 4 - 2 + c$ (M1)

$c = 7$ (A1)

$y = \frac{x^4}{4} + x^2 - x - 7$ (A1) (C6)

[6]

39. (a) $\int (1 + 3\sin(x+2))dx = x - 3\cos(x + 2) + c$ (A1)(A1)(A1)(C3)

***Notes:** Award A1 for x, A1 for –cos (x + 2) A1 for coefficient 3, ie A1 A1 for the second term, which may be written as +3(–cos (x + 2))*
*Do **not** penalize the omission of c.*

(b) $1 + 3\sin(x + 2) = 0$ (M1)

$\sin(x + 2) = -\frac{1}{3}$

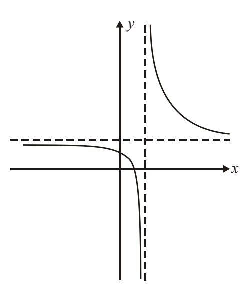
$x + 2 = -0.3398, \pi + 0.3398, \ldots$ (A1)

$x = -2.3398, 1.4814, \ldots$

Required value of $x = 1.48$ (A1) (C3)

[6]

40. (a)



(A1)(A1)2

Note: *Award (A1) for a second branch in approximately the correct position, and (A1) for the second branch having positive x and y intercepts. Asymptotes need not be drawn.*

(b) (i) x-intercept = $\frac{1}{2}\left(\text{Accept}\left(\frac{1}{2},0\right), x=\frac{1}{2}\right)$ (A1)

y-intercept = 1 (Accept (0, 1), y = 1) (A1)

(ii) horizontal asymptote $y = 2$ (A1)

vertical asymptote $x = 1$ (A1)4

(c) (i) $f'(x) = 0 - (x-1)^{-2}\left(=\frac{-1}{(x-1)^2}\right)$ (A2)

(ii) no maximum / minimum points.

since $\frac{-1}{(x-1)^2} \neq 0$ (R1)3

(d) (i) $2x + \ln(x-1) + c$ (accept $\ln|x-1|$) (A1)(A1)(A1)

(ii) $A = \int_2^4 f(x)\mathrm{d}x\left(\text{Accept}\int_2^4\left(2+\frac{1}{x-1}\right)\mathrm{d}x, [2x+\ln(x-1)]_2^4\right)$ (M1)(A1)

Notes: *Award (A1) for* ***both*** *correct limits.*
Award (M0)(A0) for an incorrect function.

(iii) $A = [2x+\ln(x-1)]_2^4$

$= (8 + \ln 3) - (4 + \ln 1)$ (M1)

$= 4 + \ln 3 (= 5.10$, to 3 sf) (A1) (N2)7

[16]

41. $f(x) = -\frac{1}{2}\mathrm{e}^{-2x} - \ln(1-x) + c$ (M1)(A1)(A1)

Substituting $4 = -\frac{1}{2}\mathrm{e}^{-2(0)} - \ln(1-0) + c$ $\left(\text{or } 4 = -\frac{1}{2} - \ln 1 + c\right)$ (M1)

$c = 4.5$ (A1)

$f(x) = -\frac{1}{2}\mathrm{e}^{-2x} - \ln(1-x) + 4.5$ (A1)(C2)(C2)(C2)

[6]

42. (a) (i) 16 (A2)(C2)

(ii) $\int_0^3 f(x)\mathrm{d}x + \int_0^3 2\,\mathrm{d}x$ (or appropriate sketch) (M1)

$= 14$ (A1)(C2)

(b) $\int_c^d f(x-2)\,\mathrm{d}x = 8$

$c = 2, d = 5$ (A2)(C2)

[6]

43. (a) (i) $a = 1 - \pi$ (accept $(1-\pi, 0)$) (A1)

(ii) $b = 1 + \pi$ (accept $(1+\pi, 0)$) (A1) 2

(b) (i) $\int_{-2.14}^{1} h(x)\mathrm{d}x - \int_1^2 h(x)\mathrm{d}x$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\mathrm{d}x + \left|\int_1^2 h(x)\mathrm{d}x\right|$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\mathrm{d}x + \int_2^1 h(x)\mathrm{d}x$ (M1)(A1)(A1)

(ii) $5.141... - (-0.1585...)$

$= 5.30$ (A2) 5

(c) (i) $y = 0.973$ (A1)
(ii) $-0.240 < k < 0.973$ (A3) 4

[11]

44. (a) $y = 0$ (A1)1

(b) $f'(x) = \dfrac{-2x}{(1+x^2)^2}$ (A1)(A1)(A1) 3

(c) $\dfrac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) (M1)

$6x^2 - 2 = 0$ (A1)

$x = \pm\sqrt{\dfrac{1}{3}}$ (A1)

$x = \dfrac{-1}{\sqrt{3}} (= -0.577)$ (A1) (N4) 4

(d) $\int_{-0.5}^{0.5} \dfrac{1}{1+x^2}\mathrm{d}x \left(= 2\int_0^{0.5} \dfrac{1}{1+x^2}\mathrm{d}x = 2\int_{-0.5}^{0} \dfrac{1}{1+x^2}\mathrm{d}x \right)$ (A1)(A1) 2

[10]

45. (a) Using the chain rule (M1)
$f'(x)$ =(2 cos (5x–3)) 5 (= 10 cos (5x – 3)) A1
$f''(x)$ = –(10 sin (5x–3)) 5
= –50 sin (5x – 3) A1A1 4

***Note**: Award (A1) for sin (5x – 3), (A1) for –50.*

(b) $\int f(x)\mathrm{d}x = \dfrac{2}{5}\cos(5x-3) + c$ A1A1 2

***Note**: Award (A1) for cos (5x–3), (A1) for* $-\dfrac{2}{5}$.

[6]

46. (a) $f'(x) = 5(3x+4)^4 \times 3 \ \left(= 15(3x+4)^4\right)$ (A1)(A1)(A1)(C3)

(b) $\int (3x+4)^5 \mathrm{d}x = \dfrac{1}{3} \times \dfrac{1}{6}(3x+4)^6 + c \left(= \dfrac{(3x+4)^6}{18} + c \right)$ (A1)(A1)(A1) (C3)

[6]

47. Attempting to integrate. (M1)
$y = x^3 - 5x + c$ (A1)(A1)(A1)
substitute (2, 6) to find c $\left(6 = 2^3 - 5(2) + c\right)$ (M1)
$c = 8$ (A1)
$y = x^3 - 5x + 8$ (Accept $x^3 - 5x + 8$) (C6)

[6]

48. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(f(x) + g(x)\right) = f'(x) + g'(x) \ \left(= f'(4) + g'(4)\right)$ (M1)
$= 7 + 4$
$= 11$ (A1)(C2)

(b) $\int_1^3 \left(g'(x) + 6\right)\mathrm{d}x = \left[g(x)\right]_1^3 + \left[6x\right]_1^3$ (A1)(A1)
$= \left(g(3) - g(1)\right) + (18 - 6) \ \left(= (2-1) + 12\right)$ (A1)
$= 13$ (A1)(C4)

[6]

49. Using $\int \dfrac{1}{x} = \ln x$ (may be implied) (M1)

$\int_3^k \frac{1}{x-2}\,dx = [\ln(x-2)]_3^k$ (A1)

$= \ln(k-2) - \ln 1$ (A1)(A1)

$\ln(k-2) - \ln 1 = \ln 7$

$k - 2 = 7$ (A1)

$k = 9$ (A1) (C6)

[6]

50.

Note: *There are many approaches possible. However, there must be some evidence of their method.*

Area = $\int_0^k \sin 2x\,dx$ (must be seen somewhere) (A1)

Using area = 0.85 (must be seen somewhere) (M1)

EITHER

Integrating $\left[\frac{-1}{2}\cos 2x\right]_0^k$

$\left(= \frac{-1}{2}\cos 2k + \frac{1}{2}\cos 0\right)$ (A1)

Simplifying $\frac{-1}{2}\cos 2k + 0.5$ (A1)

Equation $\frac{-1}{2}\cos 2k + 0.5 = 0.85$ ($\cos 2k = -0.7$)

OR

Evidence of using trial and error on a GDC (M1)(A1)

Eg $\int_0^{\frac{\pi}{2}} \sin 2x\,dx = 0.5$, $\frac{\pi}{2}$ too small etc

OR

Using GDC and solver, starting with $\int_0^k \sin 2x\,dx - 0.85 = 0$ (M1)(A1)

THEN

$k = 1.17$ (A2) (N3)

[6]