SL Probability Practice 1

1. In a survey, 100 students were asked "do you prefer to watch television or play sport?" Of the 46 boys in the survey, 33 said they would choose sport, while 29 girls made this choice.

	Boys	Girls	Total
Television			
Sport	33	29	
Total	46		100

By completing this table or otherwise, find the probability that

(a) a student selected at random prefers to watch television;

(b) a student prefers to watch television, given that the student is a boy.

Working:

Answers:

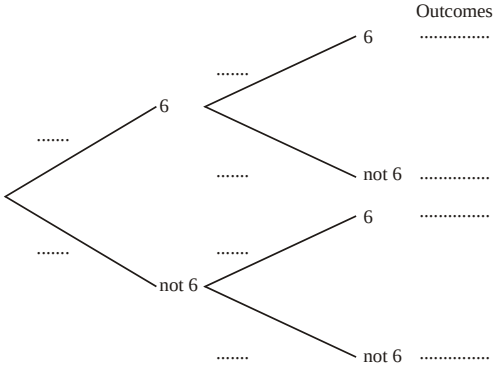
(a) .....

(b) .....

(Total 4 marks)

2. Two ordinary, 6-sided dice are rolled and the total score is noted.

(a) Complete the tree diagram by entering probabilities and listing outcomes.



(b) Find the probability of getting one or more sixes.

Working:

Answer:

(b) .....

(Total 4 marks)

3. In a survey of 200 people, 90 of whom were female, it was found that 60 people were unemployed, including 20 males.

(a) Using this information, complete the table below.

	Males	Females	Totals
Unemployed			
Employed			
Totals			200

(b) If a person is selected at random from this group of 200, find the probability that this person is

(i) an unemployed female;

(ii) a male, given that the person is employed.

Working:

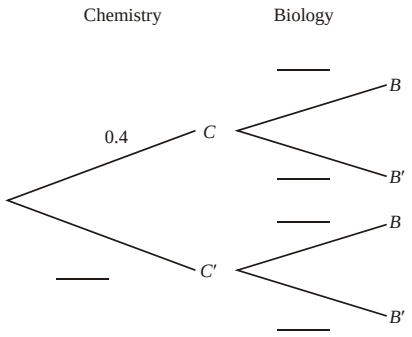
Answers:
(b) (i) .....
(ii) .....

(Total 4 marks)

4. The events B and C are dependent, where C is the event "a student takes Chemistry", and B is the event "a student takes Biology". It is known that

$$P(C) = 0.4,\ P(B \mid C) = 0.6,\ P(B \mid C') = 0.5.$$

(a) Complete the following tree diagram.



(b) Calculate the probability that a student takes Biology.

(c) Given that a student takes Biology, what is the probability that the student takes Chemistry?

Working:

Answers:
(b) .....
(c) .....

(Total 4 marks)

5. A bag contains 10 red balls, 10 green balls and 6 white balls. Two balls are drawn at random from the bag without replacement. What is the probability that they are of different colours?

Working:

Answer:

.....

(Total 4 marks)

6. A box contains 22 red apples and 3 green apples. Three apples are selected at random, one after the other, without replacement.

(a) The first two apples are green. What is the probability that the third apple is red?

(b) What is the probability that exactly two of the three apples are red?

Working:

Answers:

(a) .....

(b) .....

(Total 6 marks)

7. For the events A and B, $p(A) = 0.6$, $p(B) = 0.8$ and $p(A \cup B) = 1$.

Find

(a) $p(A \cap B)$;

(b) $p(A' \cup B')$.

Working:

Answers:

(a) .....

(b) .....

(Total 4 marks)

8. Two fair dice are thrown and the number showing on each is noted. The sum of these two numbers is S. Find the probability that

(a) S is less than 8;

(2)

(b) at least one die shows a 3;

(2)

(c) at least one die shows a 3, given that S is less than 8.

(3)

(Total 7 marks)

9. Dumisani is a student at IB World College.

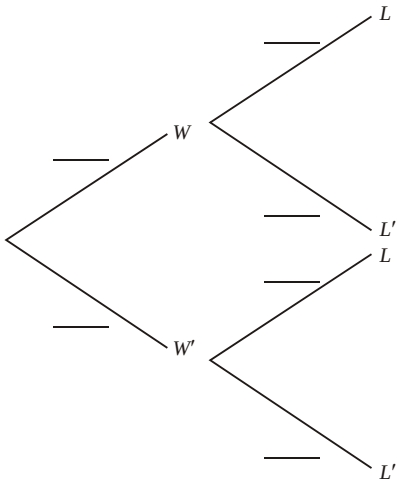
The probability that he will be woken by his alarm clock is $\frac{7}{8}$.

If he is woken by his alarm clock the probability he will be late for school is $\frac{1}{4}$.

If he is not woken by his alarm clock the probability he will be late for school is $\frac{3}{5}$.

Let *W* be the event "Dumisani is woken by his alarm clock".
Let *L* be the event "Dumisani is late for school".

(a) Copy and complete the tree diagram below.



(4)

(b) Calculate the probability that Dumisani will be late for school.

(3)

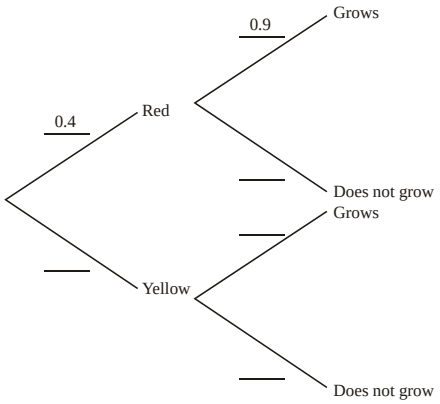
(c) Given that Dumisani is late for school what is the probability that he was woken by his alarm clock?

(4)

(Total 11 marks)

10. A packet of seeds contains 40% red seeds and 60% yellow seeds. The probability that a red seed grows is 0.9, and that a yellow seed grows is 0.8. A seed is chosen at random from the packet.

(a) Complete the probability tree diagram below.



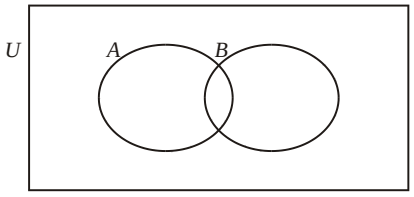
(3)

(b) (i) Calculate the probability that the chosen seed is red and grows.

(ii) Calculate the probability that the chosen seed grows.
(iii) Given that the seed grows, calculate the probability that it is red.

(7)
(Total 10 marks)

11. The following Venn diagram shows a sample space U and events A and B.



$n(U) = 36$, $n(A) = 11$, $n(B) = 6$ and $n(A \cup B)' = 21$.

(a) On the diagram, shade the region $(A \cup B)'$.

(b) Find

(i) $n(A \cap B)$;

(ii) $P(A \cap B)$.

(c) Explain why events A and B are not mutually exclusive.

Working:

Answers:

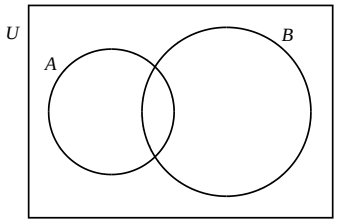
(b) (i) .....

(ii) .....

(c) .....

(Total 4 marks)

12. The following Venn diagram shows the universal set U and the sets A and B.



(a) Shade the area in the diagram which represents the set $B \cap A'$.

$n(U) = 100$, $n(A) = 30$, $n(B) = 50$, $n(A \cup B) = 65$.

(b) Find $n(B \cap A')$.

(c) An element is selected at random from U. What is the probability that this element is in $B \cap A'$?

Working:

Answers:

(b) .....

(c) .....

(Total 4 marks)

13. Consider events A, B such that $P(A) \neq 0$, $P(A) \neq 1$, $P(B) \neq 0$, and $P(B) \neq 1$.
In each of the situations (a), (b), (c) below state whether A and B are
mutually exclusive (M);
independent (I);
neither (N).

(a) $P(A|B) = P(A)$

(b) $P(A \cap B) = 0$

(c) $P(A \cap B) = P(A)$

Working:

Answers:

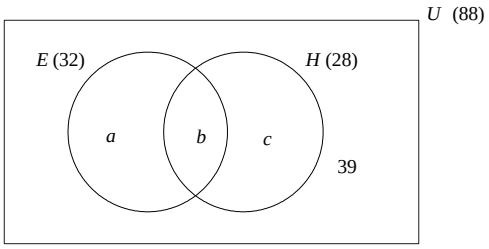
(a) .....

(b) .....

(c) .....

(Total 6 marks)

14. In a school of 88 boys, 32 study economics (E), 28 study history (H) and 39 do not study either subject. This information is represented in the following Venn diagram.



(a) Calculate the values a, b, c.

(4)

(b) A student is selected at random.

(i) Calculate the probability that he studies **both** economics and history.

(ii) Given that he studies economics, calculate the probability that he does **not** study history.

(3)

(c) A group of three students is selected at random from the school.

(i) Calculate the probability that none of these students studies economics.

(ii) Calculate the probability that at least one of these students studies economics.

(5)

(Total 12 marks)

15. Let A and B be events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{7}{8}$.

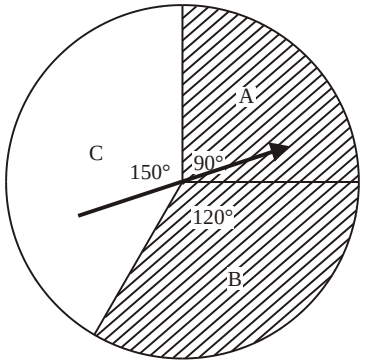
(a) Calculate $P(A \cap B)$.
(b) Calculate $P(A \mid B)$.
(c) Are the events A and B independent? Give a reason for your answer.

Working:

Answers:
(a) ……..
(b) ……..
(c) ……..

(Total 6 marks)

16. The following diagram shows a circle divided into three sectors A, B and C. The angles at the centre of the circle are 90°, 120° and 150°. Sectors A and B are shaded as shown.



The arrow is spun. It cannot land on the lines between the sectors. Let A, B, C and S be the events defined by

A: Arrow lands in sector A
B: Arrow lands in sector B
C: Arrow lands in sector C
S: Arrow lands in a shaded region.

Find
(a) $P(B)$;
(b) $P(S)$;
(c) $P(A \mid S)$.

Working:

Answers:
(a) ……..
(b) ……..
(c) ……..

(Total 6 marks)

SL Probability Practice 1 - MarkScheme

1. (a)

	Boy	Girl	Total
TV	**13**	**25**	**38**
Sport	33	29	**62**
Total	46	**54**	100

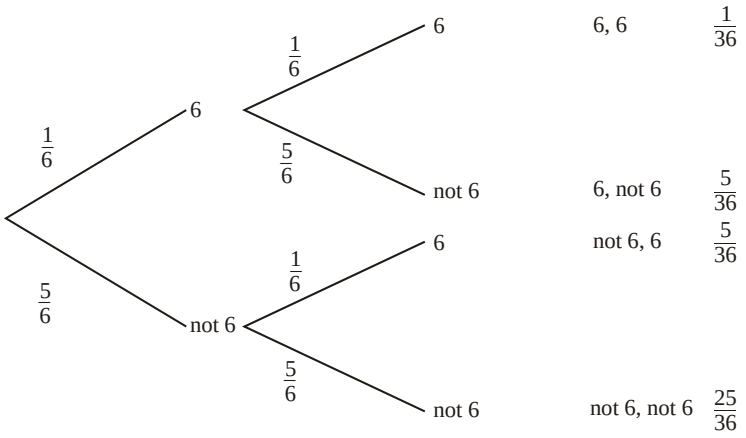
$P(TV) = \frac{38}{100}$ (A1) (C2)

(b) $P(TV \mid Boy) = \frac{13}{46}$ (= 0.283 to 3 sf) (A2) (C2)

Notes: *Award (A1) for numerator and (A1) for denominator. Accept equivalent answers.*

[4]

2. (a)



(M2) (C2)

Notes: *Award (M1) for probabilities* $\frac{1}{6}, \frac{5}{6}$ *correctly entered on diagram.*

Award (M1) for correctly listing the outcomes 6, 6; 6 not 6; not 6, 6; not 6, not 6, or the corresponding probabilities.

(b) $P(\text{one or more sixes}) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}$ **or** $\left(1-\frac{5}{6}\times\frac{5}{6}\right)$ (M1)

$= \frac{11}{36}$ (A1) (C2)

[4]

3. (a)

	Males	Females	Totals
Unemployed	**20**	**40**	**60**
Employed	**90**	**50**	**140**
Totals	**110**	**90**	200

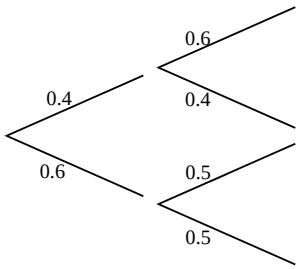
Note: *Award (A1) if at least 4 entries are correct.*
Award (A2) if all 8 entries are correct.

(b) (i) $P(\text{unemployed female}) = \frac{40}{200} = \frac{1}{5}$ (A1)

(ii) $P(\text{male I employed person}) = \frac{90}{140} = \frac{9}{14}$ (A1)

[4]

4. (a)



(A1)(C1)

(b) $P(B) = 0.4(0.6) + 0.6\,(0.5) = 0.24 + 0.30$ (M1)
$= 0.54$ (A1) (C2)

(c) $P(C \mid B) = \dfrac{P(B \cap C)}{P(B)} = \dfrac{0.24}{0.54} = \dfrac{4}{9}$ (= 0.444, 3 sf) (A1) (C1)

[4]

5. P(different colours) = 1 – [P(GG) + P(RR) + P(WW)] (M1)

$= 1 - \left(\dfrac{10}{6} \times \dfrac{9}{25} + \dfrac{10}{26} \times \dfrac{9}{25} + \dfrac{6}{26} \times \dfrac{5}{25}\right)$ (A1)

$= 1 - \left(\dfrac{210}{650}\right)$ (A1)

$= \dfrac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

OR

P(different colours) = P(GR) + P(RG) + P(GW) + P(WG) + P(RW) + P(WR) (A1)

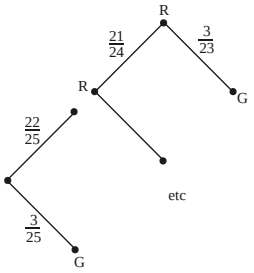
$= 4\left(\dfrac{10}{26} \times \dfrac{6}{25}\right) + 2\left(\dfrac{10}{26} \times \dfrac{10}{25}\right)$ (A1)(A1)

$= \dfrac{44}{65}$ (= 0.677, to 3 sf) (A1) (C4)

[4]

6. (a) $P = \dfrac{22}{23}$ (= 0.957 (3 sf)) (A2)(C2)

(b)



(M1)

OR

P = P (RRG) + P (RGR) + P (GRR) (M1)

$\dfrac{22}{25} \times \dfrac{21}{24} \times \dfrac{3}{23} + \dfrac{22}{25} \times \dfrac{3}{24} \times \dfrac{21}{23} + \dfrac{3}{25} \times \dfrac{22}{24} \times \dfrac{21}{23}$ (M1)(A1)

$= \dfrac{693}{2300}$ (= 0.301 (3 sf)) (A1) (C4)

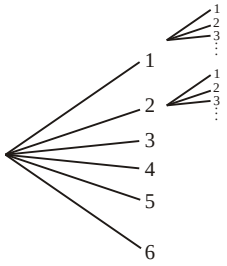
[6]

7. (a) $p(A \cap B) = 0.6 + 0.8 - 1$ (M1)
$= 0.4$ (A1)(C2)

(b) $p(\complement A \cup \complement B) = p(\complement(A \cap B)) = 1 - 0.4$ (M1)
$= 0.6$ (A1) (C2)

[4]

8. Sample space ={(1, 1), (1, 2) ... (6, 5), (6, 6)}
(This may be indicated in other ways, for example, a grid or a tree diagram, partly or fully completed)



(a) $P(S < 8) = \frac{6+5+4+3+2+1}{36}$ (M1)

$= \frac{7}{12}$ (A1)

OR

$P(S < 8) = \frac{7}{12}$ (A2)

(b) $P(\text{at least one 3}) = \frac{1+1+6+1+1+1}{36}$ (M1)

$= \frac{11}{36}$ (A1)

OR

$P(\text{at least one 3}) = \frac{11}{36}$ (A2)

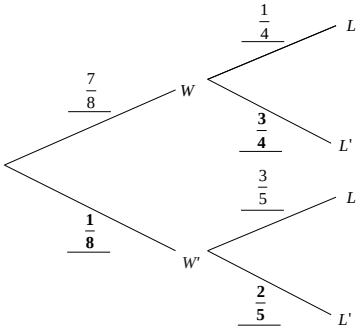
(c) $P(\text{at least one 3} \mid S < 8) = \frac{P(\text{at least one 3} \cap S < 8)}{P(S < 8)}$ (M1)

$= \frac{7/36}{7/12}$ (A1)

$= \frac{1}{3}$ (A1)

[7]

9. (a)



(A1)(A1)(A1)(A1) 4

Note: *Award (A1) for the given probabilities* $\left(\frac{7}{8}, \frac{1}{4}, \frac{3}{5}\right)$ *in the correct positions, and (A1) for each* ***bold*** *value.*

(b) Probability that Dumisani will be late is $\frac{7}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{5}$ (A1)(A1)

$= \frac{47}{160}$ (0.294) (A1) (N2) 3

(c) $P(W|L) = \frac{P(W \cap L)}{P(L)}$

$P(W \cap L) = \frac{7}{8} \times \frac{1}{4}$ (A1)

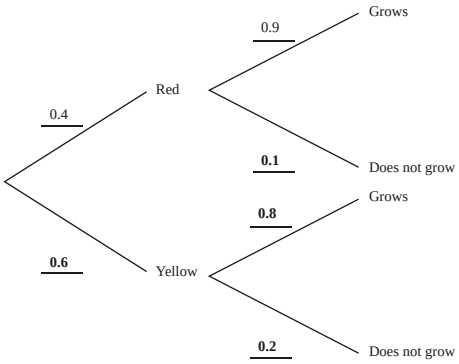
$P(L) = \frac{47}{160}$ (A1)

$P(W|L) = \frac{\frac{7}{32}}{\frac{47}{160}}$ (M1)

$= \frac{35}{47}$ (= 0.745) (A1) (N3) 4

[11]

10. (a)



(A3)(N3) 3

(b) (i) 0.4×0.9 (A1)

$= 0.36$ (A1) (N2)

(ii) $0.36 + 0.6 \times 0.8$ $(= 0.36 + 0.48)$ (A1)

$= 0.84$ (A1) (N1)

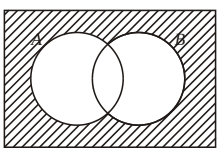
(iii) $\frac{P(\text{red} \cap \text{grows})}{P(\text{grows})}$ (may be implied) (M1)

$= \frac{0.36}{0.84}$ (A1)

$= 0.429 \left(\frac{3}{7}\right)$ (A1) (N2) 7

[10]

11. (a)



(A1)(C1)

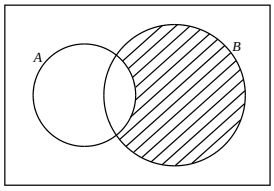
(b) (i) $n(A \cap B) = 2$ (A1) (C1)

(ii) $P(A \cap B) = \frac{2}{36}\left(\text{or}\frac{1}{18}\right)$ (allow **ft** from (b)(i)) (A1) (C1)

(c) $n(A \cap B) \neq 0$ (or equivalent) (R1) (C1)

[4]

12. (a) U



(A1)(C1)

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$65 = 30 + 50 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 15$ (may be on the diagram) (M1)

$n(B \cap A') = 50 - 15 = 35$ (A1) (C2)

(c) $P(B \cap A') = \frac{n(B \cap A')}{n(U)} = \frac{35}{100} = 0.35$ (A1) (C1)

[4]

13. (a) Independent (I) (C2)

(b) Mutually exclusive (M) (C2)

(c) Neither (N) (C2)

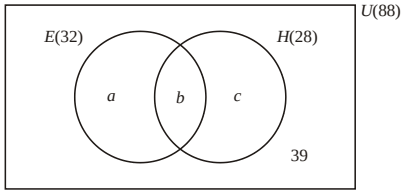
Note: *Award part marks if the candidate shows understanding of I and/or M*

eg I $P(A \cap B) = P(A)P(B)$ *(M1)*

M $P(A \cup B) = P(A) + P(B)$ *(M1)*

[6]

14. (a)



$n(E \cup H) = a + b + c = 88 - 39 = 49$ (M1)

$n(E \cup H) = 32 + 28 - b = 49$

$60 - 49 = b = 11$ (A1)

$a = 32 - 11 = 21$ (A1)

$c = 28 - 11 = 17$ (A1) 4

Note: *Award (A3) for correct answers with no working.*

(b) (i) $P(E \cap H) = \frac{11}{88} = \frac{1}{8}$ (A1)

(ii) $P(H' \mid E) = \frac{P(H' \cap E)}{P(E)} = \frac{\frac{21}{88}}{\frac{32}{88}}$ (M1)

$= \frac{21}{32}$ (= 0.656) (A1)

OR

Required probability = $\frac{21}{32}$ (A1)(A1) 3

(c) (i) P(none in economics) = $\frac{56\times55\times54}{88\times87\times86}$ (M1)(A1)

= 0.253 (A1)

Notes: *Award (M0)(A0)(A1)(ft) for* $\left(\frac{56}{88}\right)^3$ *= 0.258.*

Award no marks for $\frac{56\times55\times54}{88\times88\times88}$.

(ii) P(at least one) = 1 – 0.253 (M1)

= 0.747 (A1)

OR

$3\left(\frac{32}{88}\times\frac{56}{87}\times\frac{55}{86}\right)+3\left(\frac{32}{88}\times\frac{31}{87}\times\frac{56}{86}\right)+\frac{32}{88}\times\frac{31}{87}\times\frac{30}{86}$ (M1)

= 0.747 (A1) 5

[12]

15. (a) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ (M1)

$P(A\cap B) = \frac{1}{2}+\frac{3}{4}-\frac{7}{8}$

$= \frac{3}{8}$ (A1)(C2)

(b) $P(A|B) = \frac{P(A\cap B)}{P(B)}\left(=\frac{\frac{3}{8}}{\frac{3}{4}}\right)$ (M1)

$= \frac{1}{2}$ (A1) (C2)

(c) Yes, the events are independent (A1) (C1)

EITHER

$P(A|B) = P(A)$ (R1) (C1)

OR

$P(A\cap B) = P(A)P(B)$ (R1) (C1)

[6]

16. (a) $\frac{120}{360}\left(=\frac{1}{3}=0.333\right)$ (A1)(A1)(C2)

(b) $\frac{90+120}{360}\left(=\frac{210}{360}=\frac{7}{12}=0.583\right)$ (A2) (C2)

(c) $\frac{90}{210}\left(=\frac{3}{7}=0.429\right)$ $\left(\text{Accept } \frac{\frac{1}{4}}{\frac{7}{12}}\right)$ (A1)(A1)(C2)

[6]