Circular Functions and Trig - Practice Problems (08 & 09)

1. The circle shown has centre O and radius 3.9 cm.

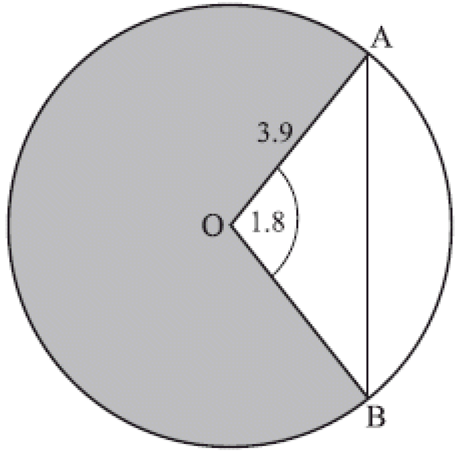


diagram not to scale

Points A and B lie on the circle and angle AOB is 1.8 radians.

(a) Find AB.

(3)

(b) Find the area of the shaded region.

(4)

(Total 7 marks)

2. The following diagram shows a circle with centre O and radius 4 cm.

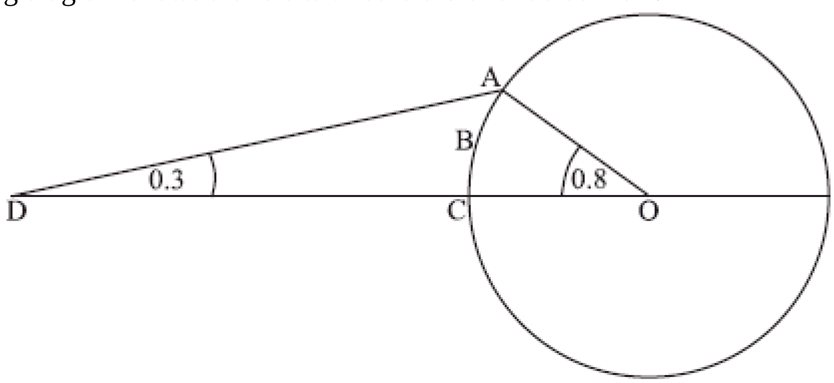


diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).
Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

(a) Find AD.

(3)

(b) Find OD.

(4)

(c) Find the area of sector OABC.

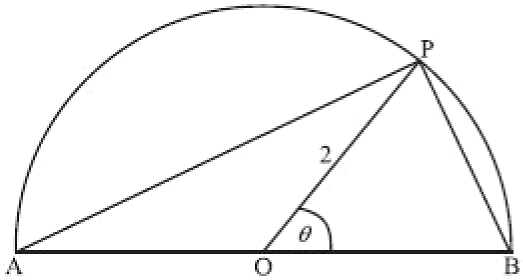
(2)

(d) Find the area of region ABCD.

(4)

(Total 13 marks)

3. The following diagram shows a semicircle centre O, diameter [AB], with radius 2.
Let P be a point on the circumference, with $P\hat{O}B = \theta$ radians.



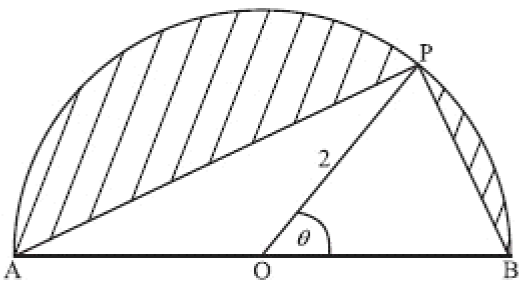
(a) Find the area of the triangle OPB, in terms of θ.

(2)

(b) Explain why the area of triangle OPA is the same as the area triangle OPB.

(3)

Let S be the total area of the two segments shaded in the diagram below.



(c) Show that $S = 2(\pi - 2 \sin \theta)$.

(3)

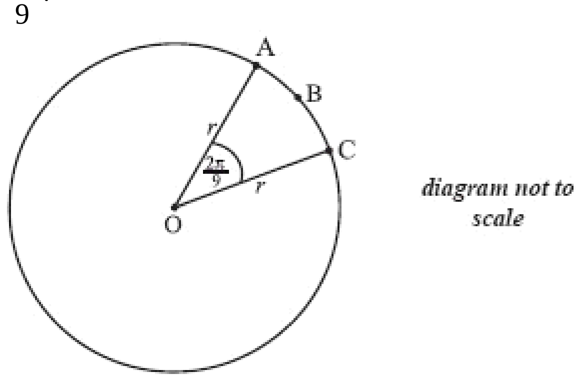
(d) Find the value of θ when S is a local minimum, justifying that it is a minimum.

(8)

(e) Find a value of θ for which S has its greatest value.

(2)
(Total 18 marks)

4. The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $A\hat{O}C = \frac{2\pi}{9}$.



(a) Find the value of r.

(2)

(b) Find the perimeter of sector OABC.

(2)

(c) Find the area of sector OABC.

(2)
(Total 6 marks)

5. The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

(a) Find

(i) $\overrightarrow{PQ}$;

(ii) $\overrightarrow{PR}$.

(3)

(b) Show that $\cos R\hat{P}Q = \frac{1}{2}$.

(7)

(c) (i) Find $\sin R\hat{P}Q$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

(6)

(Total 16 marks)

6. The diagram below shows a triangle ABD with AB = 13 cm and AD = 6.5 cm.
Let C be a point on the line BD such that BC = AC = 7 cm.

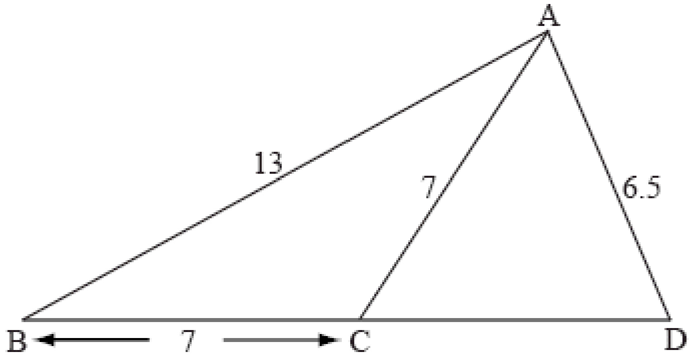


diagram not to scale

(a) Find the size of angle ACB.

(3)

(b) Find the size of angle CAD.

(5)

(Total 8 marks)

7. The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $P\hat{Q}R$ is 75°.

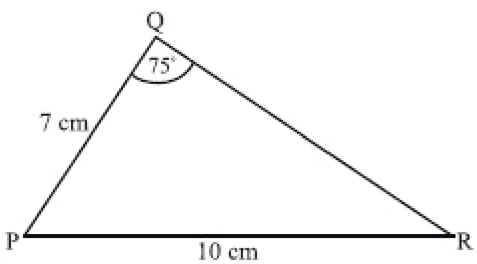


diagram not to scale

(a) Find $P\hat{Q}R$.

(3)

(b) Find the area of triangle PQR.

(3)

(Total 6 marks)

8. A ship leaves port A on a bearing of 030°. It sails a distance of 25 km to point B.
At B, the ship changes direction to a bearing of 100°. It sails a distance of 40 km to reach point C. This information is shown in the diagram below.

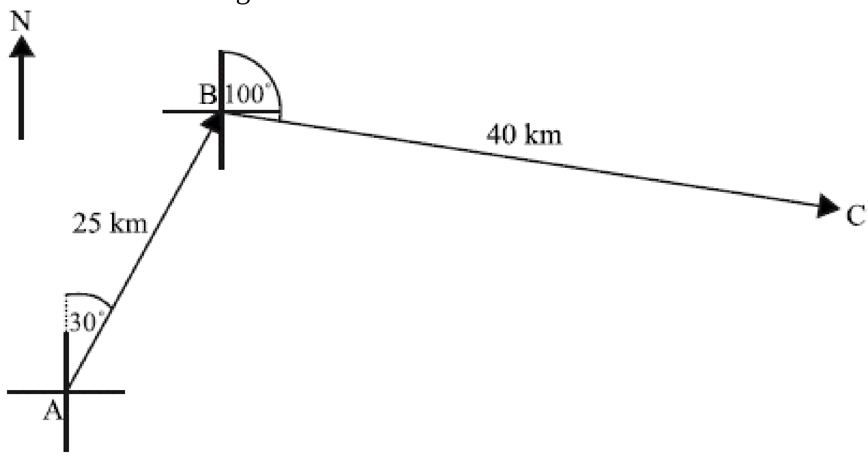


diagram not to scale

A second ship leaves port A and sails directly to C.

(a) Find the distance the second ship will travel.

(4)

(b) Find the bearing of the course taken by the second ship.

(3)
(Total 7 marks)

9. Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

(a) Find an expression for $h(x)$.

(3)

(b) Write down the period of h.

(1)

(c) Write down the range of h.

(2)
(Total 6 marks)

10. Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

(a) Sketch the graph of f.

(3)

(b) Write down
(i) the amplitude;
(ii) the period;
(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p\sin(qx + r)$.

(3)

(d) Write down one value of x such that $f'(x) = 0$.

(2)

(e) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(f) Let $g(x) = \ln(x + 1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x.

(5)
(Total 18 marks)

11. Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \le x \le 9$.

(a) On the same diagram, sketch the graphs of f and g.

(3)

(b) Consider the graph of f. Write down

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

(4)

(c) Consider the graph of g. Write down

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry.

(3)

(d) Let R be the region enclosed by the graphs of f and g. Find the area of R.

(5)

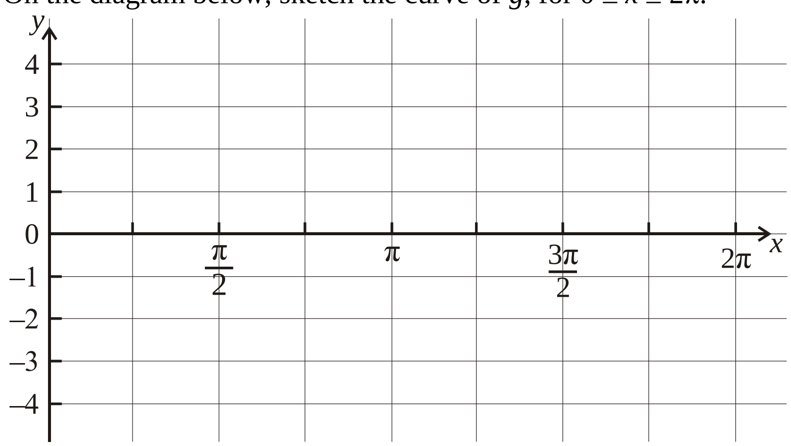
(Total 15 marks)

12. Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g.

(1)

(b) On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$.



(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$.

(2)

(Total 6 marks)

13. Let $f : x \mapsto \sin^3 x$.

(a) (i) Write down the range of the function f.

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

(5)

(b) Find $f'(x)$, giving your answer in the form $a \sin^p x \cos^q x$ where $a, p, q \in \mathbb{Z}$.

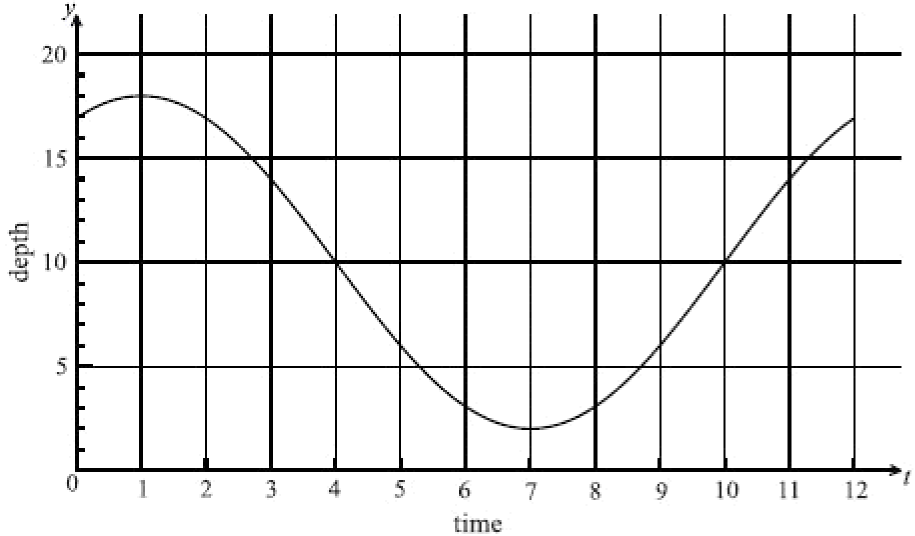
(2)

(c) Let $g(x) = \sqrt{3} \sin x (\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis.

(7)

(Total 14 marks)

14. The following graph shows the depth of water, y metres, at a point P, during one day. The time t is given in hours, from midnight to noon.



(a) Use the graph to write down an estimate of the value of t when

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

(3)

(b) The depth of water can be modelled by the function $y = A \cos (B (t - 1)) + C$.

(i) Show that $A = 8$.

(ii) Write down the value of C.

(iii) Find the value of B.

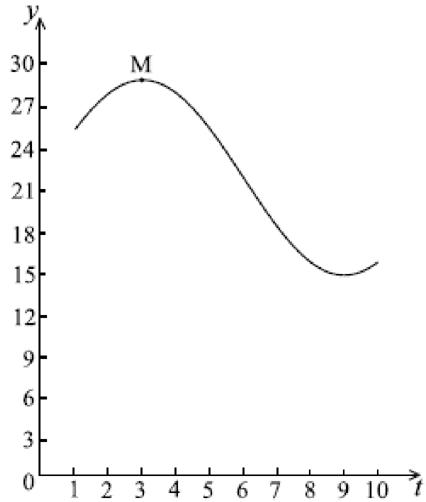
(6)

(c) A sailor knows that he cannot sail past P when the depth of the water is less than 12 m. Calculate the values of t between which he cannot sail past P.

(2)

(Total 11 marks)

15. Let $f(t) = a \cos b(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

(7)

The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′.

(2)

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$.

(4)

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f.

(3)

(Total 16 marks)

16. Solve $\cos 2x - 3 \cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \leq x \leq 2\pi$.

(Total 7 marks)

17. Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\frac{\pi}{2} < x < \pi$.

(a) Show that $f(x) = \sin x$.

(2)

(b) Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$.

(5)

(Total 7 marks)

18. Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \leq x \leq \pi$. Given that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

(Total 6 marks)

Circular Functions and Trig - Practice Problems (08 & 09) MarkScheme

1. (a) **METHOD 1**

choosing cosine rule (M1)

substituting correctly A1

e.g. AB = $\sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$

AB = 6.11(cm) A1 N2

METHOD 2

evidence of approach involving right-angled triangles (M1)

substituting correctly A1

e.g. sin 0.9 = $\frac{x}{3.9}, \frac{1}{2}$AB = 3.9 sin 0.9

AB = 6.11 (cm) A1 N2

METHOD 3

choosing the sine rule (M1)

substituting correctly A1

e.g. $\frac{\sin 0.670...}{3.9} = \frac{\sin 1.8}{\text{AB}}$

AB = 6.11 (cm) A1 N2

(b) **METHOD 1**

reflex $\text{A}\hat{\text{O}}\text{B}$ = 2π – 1.8 (= 4.4832) (A2)

correct substitution $A = \frac{1}{2}(3.9)^2(4.4832...)$ A1

area = 34.1 (cm^2) A1 N2

METHOD 2

finding area of circle $A = \pi(3.9)^2$ (= 47.78...) (A1)

finding area of (minor) sector $A = \frac{1}{2}(3.9)^2(1.8)$ (= 13.68...) (A1)

subtracting M1

e.g. $\pi(3.9)^2 - 0.5(3.9)^2(1.8)$, 47.8 – 13.7

area = 34.1 (cm^2) A1 N2

METHOD 3

finding reflex $\text{A}\hat{\text{O}}\text{B}$ = 2π – 1.8 (= 4.4832) (A2)

finding proportion of total area of circle A1

e.g. $\frac{2\pi - 1.8}{2\pi} \times \pi(3.9)^2, \frac{\theta}{2\pi} \times \pi r^2$

area = 34.1 (cm) A1 N2

[7]

2. (a) choosing sine rule (M1)

correct substitution A1

e.g. $\frac{\text{AD}}{\sin 0.8} = \frac{4}{\sin 0.3}$

AD = 9.71 (cm) A1 N2

(b) **METHOD 1**

finding angle OAD = π – 1.1 = (2.04) (seen anywhere) (A1)

choosing cosine rule (M1)

correct substitution A1

e.g. $\text{OD}^2 = 9.71^2 + 4^2 - 2 \times 9.71 \times 4 \times \cos(\pi - 1.1)$

OD = 12.1 (cm) A1 N3

METHOD 2

finding angle OAD = π – 1.1 = (2.04) (seen anywhere) (A1)

choosing sine rule (M1)
correct substitution A1

e.g. $\frac{\text{OD}}{\sin(\pi - 1.1)} = \frac{9.71}{\sin 0.8} = \frac{4}{\sin 0.3}$

OD = 12.1 (cm) A1 N3

(c) correct substitution into area of a sector formula (A1)

e.g. area = $0.5 \times 4^2 \times 0.8$

area = 6.4 (cm^2) A1 N2

(d) substitution into area of triangle formula OAD (M1)
correct substitution A1

e.g. $A = \frac{1}{2} \times 4 \times 12.1 \times \sin 0.8$, $A = \frac{1}{2} \times 4 \times 9.71 \times \sin 2.04$,

$A = \frac{1}{2} \times 12.1 \times 9.71 \times \sin 0.3$

subtracting area of sector OABC from area of triangle OAD (M1)

e.g. area ABCD = 17.3067 – 6.4

area ABCD = 10.9 (cm^2) A1 N2

[13]

3. (a) evidence of using area of a triangle (M1)

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2 \sin \theta$ A1 N2

(b) **METHOD 1**

$\text{P}\hat{\text{O}}\text{A} = \pi - \theta$ (A1)

area ΔOPA = $\frac{1}{2} 2 \times 2 \times \sin(\pi - \theta)$ (= $2 \sin(\pi - \theta)$) A1

since $\sin(\pi - \theta) = \sin \theta$ R1

then both triangles have the same area AG N0

METHOD 2

triangle OPA has the same height and the same base as triangle OPB R3

then both triangles have the same area AG N0

(c) area semi-circle = $\frac{1}{2} \times \pi (2)^2$ $(= 2\pi)$ A1

area Δ APB = $2 \sin \theta + 2 \sin \theta$ (= $4 \sin \theta$) A1

S = area of semicircle – area ΔAPB (= $2\pi - 4 \sin \theta$) M1

$S = 2(\pi - 2 \sin \theta)$ AG N0

(d) **METHOD 1**

attempt to differentiate (M1)

e.g. $\frac{\text{d}S}{\text{d}\theta} = -4 \cos \theta$

setting derivative equal to 0 (M1)

correct equation A1

e.g. $-4 \cos \theta = 0$, $\cos \theta = 0$, $4 \cos \theta = 0$

$\theta = \frac{\pi}{2}$ A1 N3

EITHER

evidence of using second derivative (M1)

$S''(\theta) = 4 \sin \theta$ A1

	$S''\left(\frac{\pi}{2}\right)=4$	A1	
	it is a minimum because $S''\left(\frac{\pi}{2}\right)>0$	R1	N0
	OR		
	evidence of using first derivative	(M1)	
	for $\theta < \frac{\pi}{2}$, $S'(\theta) < 0$ (may use diagram)	A1	
	for $\theta > \frac{\pi}{2}$, $S'(\theta) > 0$ (may use diagram)	A1	
	it is a minimum since the derivative goes from negative to positive	R1	N0
	METHOD 2		
	$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum	R3	
	$4\sin\theta$ is a maximum when $\sin\theta = 1$	(A2)	
	$\theta = \frac{\pi}{2}$	A3	N3
(e)	S is greatest when $4\sin\theta$ is smallest (or equivalent)	(R1)	
	$\theta = 0$ (or π)	A1	N2

[18]

4.

(a)	evidence of appropriate approach	M1	
	e.g. $3\pi = r\frac{2\pi}{9}$		
	$r = 13.5$ (cm)	A1	N1
(b)	adding two radii plus 3π	(M1)	
	perimeter $= 27+3\pi$ (cm) $(= 36.4)$	A1	N2
(c)	evidence of appropriate approach	M1	
	e.g. $\frac{1}{2}\times 13.5^2\times\frac{2\pi}{9}$		
	area $= 20.25\pi$ (cm^2) $(= 63.6)$	A1	N1

[6]

5.

(a) (i)	evidence of approach	(M1)	
	e.g. $\overrightarrow{PQ}=\overrightarrow{PO}+\overrightarrow{OQ}$, Q – P		
	$\overrightarrow{PQ}=\begin{pmatrix}-1\\2\\1\end{pmatrix}$	A1	N2
(ii)	$\overrightarrow{PR}=\begin{pmatrix}2\\2\\4\end{pmatrix}$	A1	N1
(b)	**METHOD 1**		
	choosing correct vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$	(A1)(A1)	
	finding $\overrightarrow{PQ}\bullet\overrightarrow{PR}$, $\lvert\overrightarrow{PQ}\rvert$, $\lvert\overrightarrow{PR}\rvert$	(A1) (A1)(A1)	
	$\overrightarrow{PQ}\bullet\overrightarrow{PR} = -2 + 4 + 4\ (= 6)$		
	$\lvert\overrightarrow{PQ}\rvert=\sqrt{(-1)^2+2^2+1^2}=(\sqrt{6}),\ \lvert\overrightarrow{PR}\rvert=\sqrt{2^2+2^2+4^2}\ (=\sqrt{24})$		
	substituting into formula for angle between two vectors	M1	

e.g. $\cos R\hat{P}Q = \frac{6}{\sqrt{6}\times\sqrt{24}}$

simplifying to expression clearly leading to $\frac{1}{2}$ A1

e.g. $\frac{6}{\sqrt{6}\times 2\sqrt{6}}, \frac{6}{\sqrt{144}}, \frac{6}{12}$

$\cos R\hat{P}Q = \frac{1}{2}$ AG N0

METHOD 2

evidence of choosing cosine rule (seen anywhere) (M1)

$\overrightarrow{QR} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ A1

$\left|\overrightarrow{QR}\right| = \sqrt{18}, \left|\overrightarrow{PQ}\right| = \sqrt{6}$ and $\left|\overrightarrow{PR}\right| = \sqrt{24}$ (A1)(A1)(A1)

$\cos R\hat{P}Q = \frac{(\sqrt{6})^2 + (\sqrt{24})^2 - (\sqrt{18})^2}{2\sqrt{6}\times\sqrt{24}}$ A1

$\cos R\hat{P}Q = \frac{6+24-18}{24}\left(= \frac{12}{24}\right)$ A1

$\cos R\hat{P}Q = \frac{1}{2}$ AG N0

(c) (i) **METHOD 1**

evidence of appropriate approach (M1)

e.g. using $\sin^2 R\hat{P}Q + \cos^2 R\hat{P}Q = 1$, diagram

substituting correctly (A1)

e.g. $\sin R\hat{P}Q = \sqrt{1-\left(\frac{1}{2}\right)^2}$

$\sin R\hat{P}Q = \sqrt{\frac{3}{4}} \quad \left(= \frac{\sqrt{3}}{2}\right)$ A1 N3

METHOD 2

since $\cos\hat{P} = \frac{1}{2}, \hat{P} = 60°$ (A1)

evidence of approach

e.g. drawing a right triangle, finding the missing side (A1)

$\sin\hat{P} = \frac{\sqrt{3}}{2}$ A1 N3

(ii) evidence of appropriate approach (M1)

e.g. attempt to substitute into $\frac{1}{2}ab\sin C$

correct substitution

e.g. area = $\frac{1}{2}\sqrt{6}\times\sqrt{24}\times\frac{\sqrt{3}}{2}$ A1

area = $3\sqrt{3}$ A1 N2

[16]

6. (a) **METHOD 1**

evidence of choosing the cosine formula (M1)

correct substitution A1

e.g. $\cos A\hat{C}B = \dfrac{7^2 + 7^2 - 13^2}{2 \times 7 \times 7}$

$A\hat{C}B$ = 2.38 radians (= 136°) A1 N2

METHOD 2

evidence of **appropriate** approach involving right-angled triangles (M1)

correct substitution A1

e.g. $\sin\left(\dfrac{1}{2} A\hat{C}B\right) = \dfrac{6.5}{7}$

$A\hat{C}B$ = 2.38 radians (= 136°) A1 N2

(b) **METHOD 1**

$A\hat{C}D = \pi - 2.381$ (180 – 136.4) (A1)

evidence of choosing the sine rule in triangle ACD (M1)

correct substitution A1

e.g. $\dfrac{6.5}{\sin 0.760\ldots} = \dfrac{7}{\sin A\hat{D}C}$

$A\hat{D}C$ = 0.836... (= 47.9...°) A1

$C\hat{A}D = \pi - (0.760... + 0.836...)$ (180 – (43.5... + 47.9...))

= 1.54 (= 88.5°) A1 N3

METHOD 2

$A\hat{B}C = \dfrac{1}{2}(\pi - 2.381)$ $\left(\dfrac{1}{2}(180 - 136.4)\right)$ (A1)

evidence of choosing the sine rule in triangle ABD (M1)

correct substitution A1

e.g. $\dfrac{6.5}{\sin 0.380\ldots} = \dfrac{13}{\sin A\hat{D}C}$

$A\hat{D}C$ = 0.836... (= 47.9...°) A1

$C\hat{A}D = \pi - 0.836... - (\pi - 2.381...)$ (= 180 – 47.9... – (180 – 136.4))

= 1.54 (= 88.5°) A1 N3

Note: *Two triangles are possible with the given information. If candidate finds $A\hat{D}C$ = 2.31 (132°) leading to $C\hat{A}D$ = 0.076 (4.35°), award marks as per markscheme.*

[8]

7. (a) choosing sine rule (M1)

correct substitution $\dfrac{\sin R}{7} = \dfrac{\sin 75°}{10}$ A1

sin R = 0.676148...

$P\hat{R}Q$ = 42.5° A1 N2

(b) P = 180 – 75 – R

P = 62.5 (A1)

substitution into any correct formula A1

e.g. area Δ PQR = $\dfrac{1}{2} \times 7 \times 10 \times \sin$ (their P)

= 31.0 (cm^2) A1 N2

[6]

8. (a) finding $\hat{ABC} = 110°$ (= 1.92 radians) (A1)

evidence of choosing cosine rule (M1)

e.g. $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos \hat{ABC}$

correct substitution A1

e.g. $AC^2 = 25^2 + 40^2 - 2(25)(40)\cos 110°$

AC = 53.9 (km) A1 N3

(b) **METHOD 1**

correct substitution into the sine rule A1

e.g. $\frac{\sin B\hat{A}C}{40} = \frac{\sin 110°}{53.9}$

$B\hat{A}C = 44.2°$ A1

bearing = 074° A1 N1

METHOD 2

correct substitution into the cosine rule A1

e.g. $\cos B\hat{A}C = \frac{40^2 - 25^2 - 53.9^2}{-2(25)(53.9)}$

$B\hat{A}C = 44.3°$ A1

bearing = 074° A1 N1

[7]

9. (a) attempt to form any composition (even if order is reversed) (M1)

correct composition $h(x) = g\left(\frac{3x}{2}+1\right)$ (A1)

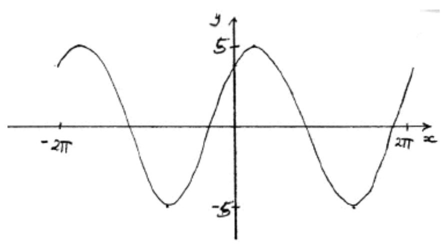
$h(x) = 4\cos\left(\frac{\frac{3x}{2}+1}{3}\right) - 1$ $\left(4\cos\left(\frac{1}{2}x+\frac{1}{3}\right)-1, 4\cos\left(\frac{3x+2}{6}\right)-1\right)$ A1 N3

(b) period is 4π (12.6) A1 N1

(c) range is $-5 \le h(x) \le 3$ ([–5, 3]) A1A1 N2

[6]

10. (a)



A1A1A1 N3

Note: *Award A1 for approximately sinusoidal shape, A1 for end points approximately correct, (–2π, 4), (2π, 4) A1 for approximately correct position of graph, (y-intercept (0, 4) maximum to right of y-axis).*

(b) (i) 5 A1 N1

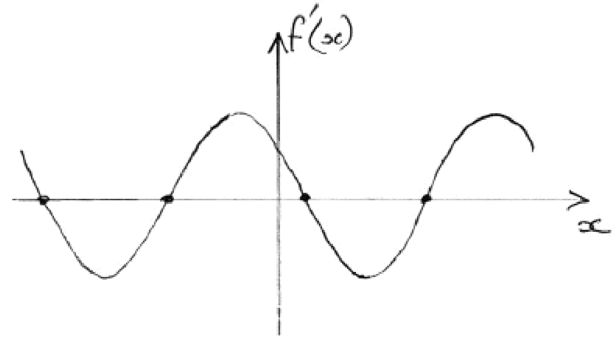
(ii) 2π (6.28) A1 N1

(iii) –0.927 A1 N1

(c) $f(x) = 5\sin(x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) A1A1A1 N3

(d) evidence of correct approach (M1)

e.g. max/min, sketch of $f'(x)$ indicating roots



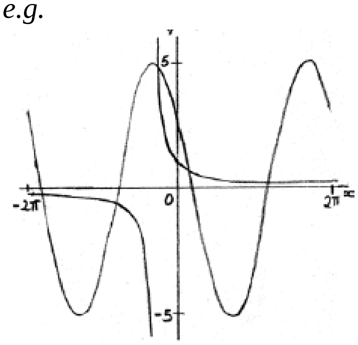
one 3 s.f. value which rounds to one of –5.6, –2.5, 0.64, 3.8 A1 N2

(e) $k = -5$, $k = 5$ A1A1 N2

(f) **METHOD 1**

graphical approach (but must involve derivative functions) M1

e.g.



each curve A1A1

$x = 0.511$ A2 N2

METHOD 2

$g'(x) = \dfrac{1}{x+1}$ A1

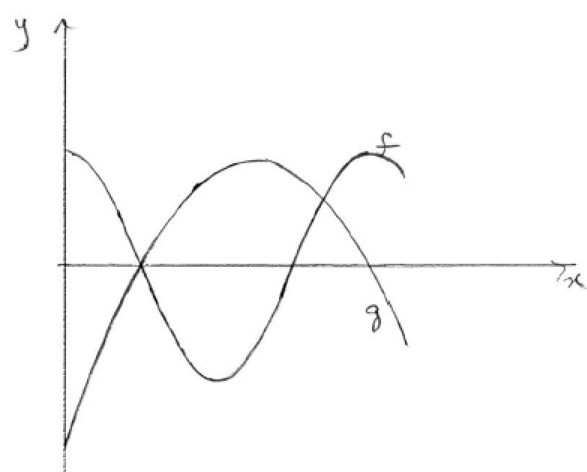
$f'(x) = 3\cos x - 4\sin x \quad (5\cos(x + 0.927))$ A1

evidence of attempt to solve $g'(x) = f'(x)$ M1

$x = 0.511$ A2 N2

[18]

11. (a)



A1A1A1 N3

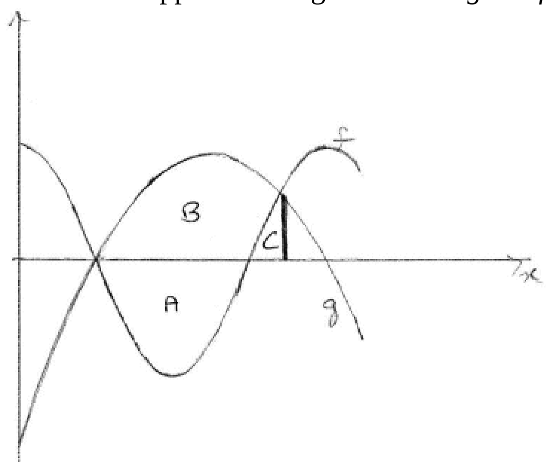
Note: *Award A1 for f being of sinusoidal shape, with 2 maxima and one minimum,*
A1 for g being a parabola opening down,
A1 for ***two*** *intersection points in approximately correct position.*

(b) (i) (2,0) (accept $x = 2$) A1 N1
(ii) period = 8 A2 N2
(iii) amplitude = 5 A1 N1
(c) (i) (2, 0), (8, 0) (accept $x = 2$, $x = 8$) A1A1 N1N1
(ii) $x = 5$ (must be an equation) A1 N1
(d) **METHOD 1**
intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) A1A1
evidence of approach (M1)

e.g. $\int g - f, \int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x, \int_2^{6.79}\left((-0.5x^2 + 5x - 8 - \left(5\cos\frac{\pi}{4}x\right)\right)$

area = 27.6 A2 N3

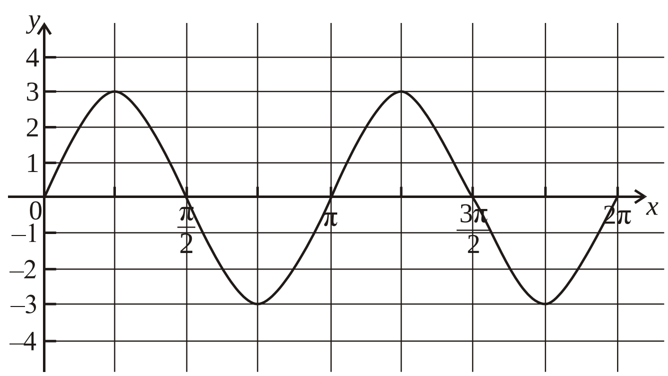
METHOD 2
intersect when $x = 2$ and $x = 6.79$ (seen anywhere) A1A1
evidence of approach using a sketch of g and f, or $g - f$. (M1)



e.g. area $A + B - C$, 12.7324 + 16.0938 – 1.18129...
area = 27.6 A2 N3

[15]

12. (a) period = π A1 N1
(b)



A1A1A1 N3

Note: *Award A1 for amplitude of 3, A1 for **their** period, A1 for a sine curve passing through (0, 0) and (0, 2π).*

(c) evidence of appropriate approach (M1)
e.g. line $y = 2$ on graph, discussion of number of solutions in the domain
4 (solutions) A1 N2

[6]

13. (a) (i) range of f is $[-1, 1]$, $(-1 \leq f(x) \leq 1)$ A2 N2

(ii) $\sin^3 x = 1 \Rightarrow \sin x = 1$ A1

justification for one solution on $[0, 2\pi]$ R1

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) A1 N1

(b) $f'(x) = 3\sin^2 x \cos x$ A2 N2

(c) using $V = \int_a^b \pi y^2 \mathrm{d}x$ (M1)

$$V = \int_0^{\frac{\pi}{2}} \pi \left(\sqrt{3} \sin x \cos^{\frac{1}{2}} x\right)^2 \mathrm{d}x$$ (A1)

$$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \, \mathrm{d}x$$ A1

$$V = \pi\left[\sin^3 x\right]_0^{\frac{\pi}{2}} \quad \left(= \pi\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0\right)\right)$$ A2

evidence of using $\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ (A1)

e.g. $\pi(1 - 0)$

$V = \pi$ A1 N1

[14]

14. (a) (i) 7 A1 N1

(ii) 1 A1 N1

(iii) 10 A1 N1

(b) (i) evidence of appropriate approach M1

e.g. $A = \frac{18-2}{2}$

$A = 8$ AG N0

(ii) $C = 10$ A2 N2

(iii) **METHOD 1**

period = 12 (A1)

evidence of using $B \times$ period $= 2\pi$ (accept 360°) (M1)

e.g. $12 = \frac{2\pi}{B}$

$B = \frac{\pi}{6}$ (accept 0.524 or 30) A1 N3

METHOD 2

evidence of substituting (M1)

e.g. $10 = 8\cos 3B + 10$

simplifying (A1)

e.g. $\cos 3B = 0$ $\left(3B = \frac{\pi}{2}\right)$

$B = \frac{\pi}{6}$ (accept 0.524 or 30) A1 N3

(c) correct answers A1A1

e.g. $t = 3.52$, $t = 10.5$, between 03:31 and 10:29 (accept 10:30) N2

[11]

15. (a) (i) attempt to substitute (M1)

e.g. $a = \frac{29-15}{2}$

$a = 7$ (accept $a = -7$) A1 N2

(ii) period = 12 (A1)

$b = \frac{2\pi}{12}$ A1

$b = \frac{\pi}{6}$ AG N0

(iii) attempt to substitute (M1)

e.g. $d = \frac{29+15}{2}$

$d = 22$ A1 N2

(iv) $c = 3$ (accept $c = 9$ from $a = -7$) A1 N1

Note: *Other correct values for c can be found,* $c = 3 \pm 12k, k \in \mathbb{Z}$.

(b) stretch takes 3 to 1.5 (A1)

translation maps (1.5, 29) to (4.5, 19) (so M′ is (4.5, 19)) A1 N2

(c) $g(t) = 7\cos\frac{\pi}{3}(t - 4.5) + 12$ A1A2A1 N4

Note: *Award* ***A1*** *for* $\frac{\pi}{3}$, ***A2*** *for 4.5,* ***A1*** *for 12.*

Other correct values for c can be found $c = 4.5 \pm 6k, k \in \mathbb{Z}$.

(d) translation $\begin{pmatrix}-3\\10\end{pmatrix}$ (A1)

horizontal stretch of a scale factor of 2 (A1)

completely correct description, in correct order A1 N3

e.g. translation $\begin{pmatrix}-3\\10\end{pmatrix}$ then horizontal stretch of a scale factor of 2

[16]

16. evidence of substituting for $\cos 2x$ (M1)

evidence of substituting into $\sin^2 x + \cos^2 x = 1$ (M1)

correct equation in terms of $\cos x$ (seen anywhere) A1

e.g. $2\cos^2 x - 1 - 3\cos x - 3 = 1$, $2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve (M1)

e.g. factorizing, quadratic formula

appropriate working A1

e.g. $(2\cos x - 5)(\cos x + 1) = 0$, $(2x - 5)(x + 1)$, $\cos x = \frac{3 \pm \sqrt{49}}{4}$

correct solutions to the equation

e.g. $\cos x = \frac{5}{2}$, $\cos x = -1$, $x = \frac{5}{2}$, $x = -1$ (A1)

$x = \pi$ A1 N4

[7]

17. (a) changing $\tan x$ into $\frac{\sin x}{\cos x}$ A1

e.g. $\sin^3 x + \cos^3 x \frac{\sin x}{\cos x}$

simplifying A1

e.g. $\sin x (\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$

$f(x) = \sin x$ AG N0

(b) recognizing $f(2x) = \sin 2x$, seen anywhere (A1)

evidence of using double angle identity $\sin (2x) = 2 \sin x \cos x$, seen anywhere (M1)

evidence of using Pythagoras with $\sin x = \frac{2}{3}$ M1

e.g. sketch of right triangle, $\sin^2 x + \cos^2 x = 1$

$\cos x = -\frac{\sqrt{5}}{3} \left(\text{accept} \frac{\sqrt{5}}{3}\right)$ (A1)

$f(2x) = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ A1

$f(2x) = -\frac{4\sqrt{5}}{9}$ AG N0

[7]

18. $e^{2x}(\sqrt{3} \sin x + \cos x) = 0$ (A1)

$e^{2x} = 0$ not possible (seen anywhere) (A1)

simplifying

e.g. $\sqrt{3}\sin x + \cos x = 0, \sqrt{3}\sin x = -\cos x, \frac{\sin x}{-\cos x} = \frac{1}{\sqrt{3}}$ A1

EITHER

$\tan x = -\frac{1}{\sqrt{3}}$ A1

$x = \frac{5\pi}{6}$ A2 N4

OR

sketch of 30°, 60°, 90° triangle with sides 1, 2, $\sqrt{3}$ A1

work leading to $x = \frac{5\pi}{6}$ A1

verifying $\frac{5\pi}{6}$ satisfies equation A1 N4

[6]